How Does Inflation "Grease the Wheels" in a Frictional Labor Market?*

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Abstract

This paper studies the role of inflation in greasing the wheels of the labor market. To do so, it unifies the theory of frictional labor markets with a theory of nominal wage adjustment. The model features worker heterogeneity, endogenous quits and layoffs, on-the-job search, and on-the-job wage renegotiation. Renegotiation costs together with the requirement of mutual agreement lead to a state-dependent process for on-the-job wage renegotiation. We parametrize the model to match important features of the distribution of wage changes within jobs and across jobs measured with administrative microdata. The new framework reproduces the anatomy of labor market dynamics during episodes of inflation surges, such as the ones observed in Argentina's 2001 inflation episode.

Keywords: Inflation, Monetary Policy, Wage Rigidity, Unemployment, Inefficient Job Separations, Quits, Layoffs, Directed Search, Commitment, Continuous-Time Methods, Variational Inequalities

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1 Introduction

The labor market plays a crucial role in the analysis of business cycle dynamics, making the study of frictions affecting this market, particularly in the form of sticky wages, essential in macroeconomic analysis. Empirically, the presence of infrequent wage adjustment is a pervasive feature observed in wage micro-data (see Grigsby *et al.*, 2021; Hazell and Taska, 2022; Blanco *et al.*, 2022a, for recent measurement of sticky wages). From a positive perspective, sticky wages are often cited as the primary friction responsible for explaining the effects of monetary shocks (Christiano *et al.*, 2005), the volatility of unemployment (Shimer, 2005a; Hall, 2005), and employment dynamics across fixed and floating exchange rate regimes (Schmitt-Grohé and Uribe, 2016). From a normative perspective, understanding the welfare consequences associated with fluctuations in unemployment due to sticky wages is crucial for the design of effective fiscal and monetary policies together with social insurance policies.

Wage adjustments may occur in three distinct scenarios: firstly, during the tenure of a current job as a result of bargaining between a firm and worker; secondly, following a job-to-job transition when a worker moves to a new firm; and thirdly, following an unemployment spell when a worker is separated from their current job and seeks new employment opportunities. Given that significant wage changes occur across jobs due to job switching, any theory of wage rigidity must explicitly incorporate a model of labor mobility. A natural starting point is the search and matching model of Pissarides (1985), which has been widely used to study the labor market dynamics of unemployment and vacancies.

This paper unifies the theory of frictional labor markets with a theory of nominal wage adjustment to analyze the aggregate fluctuations of wages and employment. The premise guiding our paper is that the analysis of these fluctuations should be grounded on a model informed by microdata on wage adjustment within and across jobs, which constitutes the primary contribution of our study.

Model Overview. We develop a new model of a frictional labor market with directed and on-the-job search. The model incorporates three key features: (i) idiosyncratic worker productivity shocks, (ii) two-sided limited commitment, and (iii) wage rigidities. The first feature, idiosyncratic shocks to worker productivity, serves as the fundamental driver of wage adjustment and labor mobility in our model. It is crucial to consider such shocks, as microdata on wages cannot be adequately explained solely by aggregate shocks; rather, they predominantly reflect idiosyncratic fluctuations in workers' productivity. The second feature introduces the possibility that workers and firms may choose to unilaterally abandon a match when it is individually advantageous but not necessarily in the best interest of the match, potentially leading to inefficiencies.

Our first contribution focuses on the analysis of wage rigidities within the job. In the model, wage rigidities result from the presence of renegotiation costs and the requirement that bargaining occurs under mutual agreement. While renegotiation costs trivially lead to wage stickiness, as menu costs do in the pricing literature, the effect of mutual agreement on wage stickiness is more nuanced and novel. On the one hand, changes in real wages can alter the distribution of rents between firms and workers, making wage adjustments primarily a redistributive concern. On the other hand, wage changes can improve match efficiency by reducing the likelihood of inefficient layoffs and quits, thereby justifying wage adjustments for efficiency reasons. Hence, the requirement of mutual agreement prevents wage changes when their primary motive is redistributive, as one of the agents would refuse to negotiate. As a result, wages remain fixed insofar the experienced cumulative productivity shocks on-the-job are relatively small. Instead, wages are renegotiated only when the worker's productivity is sufficiently high (resp. low) relative to the current wage and the match is close to the quit (resp. layoff) threshold.

While these mechanisms affect wage adjustment within a job, our model also incorporates endogenous on-the-job search effort, which shapes wage adjustment through labor mobility to a new job. In the model, search effort is high when the wage is either too low or too high relative to productivity. In the former case, workers search to move up the job ladder, a prediction shared with canonical models of on-the-job search. Instead, in the latter case, workers search on-the-job to find a new job before being laid off—a novel prediction of our framework consistent with empirical evidence (Fujita, 2010).

Taking these ingredients into account, our model predicts wage changes within and across jobs that are state-dependent, meaning that the current wage determines the probability and magnitude of future wage adjustments. Thus, our economic framework establishes a state-contingent wage Phillips curve based on micro-level labor flows.

Taking the Model to the Data. Our theory ignores many features in the microdata whenever we generate aggregate fluctuations in the labor market. For that reason, the mapping data to the model is not trivial, and we want to do the first step in this direction. The two dimensions in the data that our model abstract are: (i) transitory fluctuation in wages, and (ii) changes in wages across jobs due to permanent differences across firms. To tackle these challenges with the demanding data requirement, we use administrative employer-employee-match monthly labor income data from Argentina.

Although our administrative data may contain negligible measurement error, our model abstracts from many sources of transitory deviations from a modal or permanent wage (e.g., workers on commission or intensive margin labor supply). We construct a measure of the regular wage following the Break Test methodology developed in Blanco *et al.* (2022b). In a nutshell, the logic behind this methodology is to split a nominal wage series into two continuous subsamples and perform a statistical test of whether those subsamples were drawn from the same distribution using the Kolmogorov-Smirnov statistic. In addition, our model abstracts from wage differences arising from firm heterogeneity; in the model, productivity differences are only driven by workers' idiosyncratic shocks. Therefore, we filter the data on wage changes across firms to remove the proportion of wage changes arising from firms' fixed heterogeneity.

We calibrate our model to match labor flows and the distribution of wage changes in our microdata. Our model can generate the size and dispersion of wage changes within and across jobs. We then use the parametrized model to study the dynamics of aggregate wages and employment following an inflation surge.

Inflation Greases the Wheels of the Labor Market: Evidence Meets Model. The international macroeconomics literature provides compelling empirical evidence supporting the role of inflation in mitigating the adverse effects of wage rigidity. As Schmitt-Grohé and Uribe (2016) remark, there is a considerable quantitative difference in unemployment dynamics between countries with floating exchange rate regimes and those with fixed exchange rate regimes. A fixed change rate provides nominal stability, which leads to the propagation of productivity fluctuations to unemployment. Testing our theory in such settings is infeasible due to the absence of inflationary shocks. Instead, significant devaluations serve as unexpected and substantial shocks to the aggregate inflation rate, playing a stabilizing role during recessions by reducing real wages, as demonstrated by Blanco *et al.* (2022c). Leveraging this insight, we seize the opportunity provided by the 2001 Argentinean recession and the subsequent shift away from a fixed exchange rate regime to test our theory in a real-world laboratory setting.

Before 2001, Argentina experienced a notable decoupling of real wages from labor productivity. In the context of negative growth and macroeconomic instability, a 7% decline in marginal product of labor between 1998 and 2001 implies that real labor income should have similarly fallen by 7%. However, average real wages remained relatively stable, accompanied by a significant increase in unemployment resulting from a decrease in the job-finding rate and a surge in job separations in 2001. Following the departure from the fixed exchange rate, a substantial devaluation led to an increase in inflation to 35%, following several years of near-zero or negative inflation rates. Consequently, real wages adjusted in line with inflation, with a significant drop in job separations being the main driver of the employment recovery.

Our model replicates the dynamics of real wages and employment during this episode. We reproduce this episode with an exogenous drop in labor productivity and an unexpected inflation surge. The model matches the main driver of unemployment, which is the aggregate separation rate. Prior to the inflation shock, labor productivity experiences a prolonged decline. Due to wage rigidity, many workers face elevated wage-to-productivity ratios, prompting firms to lay off more workers than in the steady state. Following the inflation surge, the distribution of wage-to-productivity ratios shifts to lower levels, resulting in an immediate reduction in the job-separation rate, consistent with the data. It is noteworthy that our model generates unemployment dynamics without downward wage rigidity constraint at the aggregate level; instead, aggregate dynamics stem from fluctuations in the distribution of wage-to-productivity ratios across workers. This aspect is crucial since many micro-level wage changes across jobs involve wage reductions, making it challenging to justify any micro foundation for downward wage rigidity.

Literature Review. Our paper is situated within various strands of literature, primarily focusing on the interaction between inflation and the labor market. Notably, in his presidential address, Tobin (1972) highlighted the social efficiency of unemployment resulting from workers searching for suitable jobs,[1] while also recognizing that frictions in wage setting, such as wages being set in nominal terms, could create a disparity between the marginal product of labor and real wages. In particular, price inflation, "is a neutral method of making arbitrary money wage path conform to the realities of productivity growth" (Tobin, 1972, p. 13). Subsequent studies have explored Tobin's hypothesis, yielding mixed findings regarding the impact of inflation on wage adjustment. For instance, Card (1990) studied the effects of effect of nominal contracting provisions in union contracts and found that real wage changes due to unexpected price changes are associated with employment responses in the opposite direction. Additionally, Ball (1997) demonstrated that the effects of disinflation on unemployment were more pronounced in countries with heavily regulated labor markets. More recent research by Coglianese *et al.* (2021) focused on the effect of monetary shocks in Sweden, revealing that sectors with more rigid wage contracts experienced larger increases in unemployment following such shocks. We contribute to this literature by developing a model based on micro-level labor flows, where interactions between real and nominal shocks determine aggregate wages and unemployment. Our model introduces a novel mechanism, whereby the distribution of real wages serves as the state of the economy. Consequently, the impact of inflation on unemployment becomes state-dependent, underscoring the importance of considering the distribution of real wages across workers.

The process of adjustment of wages within and across jobs is crucial for aggregate labor dynamics in our model. Existing literature has studied the distribution of wage changes within jobs, revealing

[1]"The new microeconomics of job search (see Edmund Phelp et al.), is nevertheless a valuable contribution to understanding of frictional unemployment. It provides reasons why some unemployment is voluntary, and why some unemployment is socially efficient" (Tobin, 1972, p. 7).

asymmetry and a missing mass of negative wage changes at low inflation levels (Dickens *et al.*, 2007; Barattieri *et al.*, 2014; Grigsby *et al.*, 2021). Sigurdsson and Sigurdardottir (2016) observed that although nominal wage cuts are rare, their frequency rises following a large macroeconomic shock (e.g., the 2008 Great Recession), while Blanco *et al.* (2022a) documented that the distribution of nominal wage changes loses its asymmetry around zero as inflation increases. Here, we contribute to this literature by providing a model that uses the micro-evidence on wage adjustment to estimate key parameters of the wage adjustment process, akin to the approach employed in the pricing literature (see Golosov and Lucas, 2007).

Numerous papers have incorporated wage rigidity into search and matching models, often introducing it in a manner that does not disrupt existing matches and therefore does not lead to inefficient separations. For instance, Hall (2003) and Elsby *et al.* (2022) assume wage changes to prevent inefficient separations, while Gertler and Trigari (2009) consider small aggregate productivity shocks around the steady-state that do not trigger inefficient layoffs. Our micro-foundation takes a different approach. We assume that bargaining occurs under mutual agreement and is subject to renegotiation costs, which in conjunction with two-sided limited commitment can lead to inefficient flows, as empirically documented by Jäger *et al.* (2022). By doing so, we develop a framework that includes all wage adjustment opportunities observed in modern labor markets as the outcome of choices of optimizing agents while grounding them with micro-data, thereby addressing the critique put forth by Barro (1977).

2 A Model

This section describes a search and matching model to study inflation's role in greasing the labor market wheels. At its core, the model is similar to Blanco *et al.* (2022b) with idiosyncratic and aggregate shocks, sticky wages, and two-sided limited commitment. We extend this framework in two ways. First, we introduce renegotiation costs with mutual agreement. These features provide a theory of on-the-job wage renegotiation. Second, we incorporate on-the-job search following the work of Menzio and Shi (2010). Taken together, the model produces an empirically-oriented micro-foundation of aggregate wage and employment dynamics by departing from micro-labor flows and wage changes.

2.1 Environment.

Time is continuous and indexed by t. An exogenous unit mass of workers, denoted by $i \in [0, 1]$, and an endogenously determined mass of firms meet in a frictional labor market. The economy is subject to transitory and deterministic aggregate shocks to labor productivity, denoted as A_t, and the price level, denoted as P_t. Throughout the paper, we use lower-case letters to represent the natural logarithm of variables in upper-case letters. For example, a_t denotes the log aggregate productivity. Workers maximize the expected discounted utility from consumption, while firms maximize the net present value of profits. All agents discount the future at a common rate $\rho > 0$.

Preferences. Workers value an expected discounted consumption stream $\{C_{it}\}_{t=0}^{\infty}$ with risk-neutral preferences:

$$\mathbb{E}\left[\int_0^{\infty} e^{-\rho t} C_{it}\,\mathrm{d}t\right].$$

Technology. A worker's flow income depends on her employment state E_{it}, which can be either employed (h) or unemployed (u), as well as her productivity. The worker's idiosyncratic productivity z_{it} follows a Brownian motion in logs with drift γ and volatility σ—$\mathrm{d}z_{it} = \gamma\,\mathrm{d}t + \sigma\,\mathrm{d}\mathcal{W}_{it}$. While employed, a worker produces $e^{z_{it}+a_t}$ units of a homogeneous good and receives flow real income equal to the real wage $e^{w_{it}-p_t}$. While unemployed, a worker receives flow income $\tilde{B}e^{z_{it}}$ from home production.

Wage-setting Mechanism. We assume that entry wages are competitively set. Nominal wages are rigid on-the-job, and wage renegotiations entail paying a stochastic cost $\psi_t(\Delta w)e^{z_t}$ in units of output, which could be different for positive or negative wage changes. If the intended wage change is positive, with probability $1-\beta^+\,\mathrm{d}t$ the cost is $\psi_t = \infty$, and with probability $\beta^+\,\mathrm{d}t$, it is a random variable with c.d.f. $G^+(\psi)$. Similarly, for intended negative wage changes, with probability $\beta^+\,\mathrm{d}t$, the cost is a random variable with c.d.f. $G^-(\psi)$ and with probability $1-\beta^-\,\mathrm{d}t$ the cost is infinitely large. If there is mutual consent in the renegotiation process, the new wage is set according to the Nash Bargaining solution, where the worker's bargaining power is denoted by χ. The timing of the wage adjustment is the following. First, the stochastic cost is realized, and the firm decides whether or not to pay it. At that point, the renegotiation cost is a sunk cost. Then, the new nominal wage is renegotiated between the worker and the firm.

Job Creation. Workers search for jobs in a frictional labor market. Inspired by Moen (1997), search is directed and segmented across submarkets according to the log wage w and the worker's log productivity z. In each submarket $(z; w)$, firms post vacancies $\mathcal{V}$ at cost $\tilde{K}e^z$ until the marginal expected cost of vacancy posting equals its expected benefits, represented by the value of a filled job. Workers can search for jobs in a single submarket at a rate that depends on their employment status. We assume that unemployed workers search at a constant rate, which we normalize to one. Instead, employed workers can choose their search intensity s by incurring a cost of $c(s) = \frac{\mu}{1+1/\phi} s^{1+1/\phi} e^z$ in market $(z; w)$, where $\phi > 0$.

Given $\mathcal{S} = \int_i s_i \, \mathrm{d}i$ total effective units of search efficiency and $\mathcal{V}$ vacancies in a submarket $(z; w)$, a Cobb-Douglas matching function with constant returns produces $m(\mathcal{S}, \mathcal{V}) = \mathcal{S}^\alpha \mathcal{V}^{1-\alpha}$ matches, where α represents the elasticity of matches to the effective units of searchers. As a result, a worker's i job-finding rate is given by their search intensity s_i multiplied by the job-finding rate per unit of search intensity $s_i f(\theta) = s_i m/\mathcal{S} = s_i \theta^{1-\alpha}$. Similarly, a firm's job filling rate is $q(\theta) = m/\mathcal{V} = \theta^{-\alpha}$, where $\theta := \mathcal{V}/\mathcal{S}$ denotes the market tightness in submarket $(z; w)$. Thus, the job-finding rate of the unemployed is $f(\theta)$, and for an employed worker exerting a search intensity s, it is given by $s f(\theta)$.

Job Destruction. Existing matches can get exogenously dissolved according to a Poisson process with an arrival rate δ. In addition, we assume that workers and firms cannot commit to the labor contract. Therefore, at any point in time, the match can be endogenously and unilaterally dissolved by either the worker or the firm.

2.2 Recursive Equilibrium

Before delving into the equilibrium conditions, we note three properties of the model. First, due to the proportionality of unemployed workers' flow income, firms' vacancy costs, and search costs to idiosyncratic productivity e^z, the relevant state variable for both workers and firms is the log-real wage-to-productivity ratio, denoted as $\hat{w} := w - z - p$, which we simply refer to as the relative wage. Second, as search is directed, agents' equilibrium values and policies do not depend on the distribution of workers' idiosyncratic states. Third, since the path of aggregate variables is deterministic, we index all the policies and values with t.

Let $j_t(z; w)$, $u_t(z)$, and $h_t(z; w)$ denote the values of firms, unemployed and employed workers. Similarly, let $\theta(z; w)$ be the market tightness in market $(z; w)$. The properties described above imply that these values can be expressed as $j_t(z; w) = \hat{J}_t(w - z - p_t) e^z$, $u_t(z) = \hat{U}_t e^z$, and $h_t(z; w) = \hat{H}_t(w - z - p_t) e^z + \hat{U}_t$ where $\hat{J}_t(\hat{w})$ and $\hat{U}_t$ denote the firm's and the unemployed worker's values per unit of productivity, and $\hat{H}_t(\hat{w})$ represents the value of an employed worker per unit of productivity net of the unemployment value. Similarly, let $\theta_t(z; w) = \hat{\theta}_t(w - z - p_t)$ be the market tightness in market $(w - z - p_t)$. These properties regarding the value functions similarly extend to agents' policy functions. For instance, if $w^*_{u,t}(z)$ and $w^*_{jj,t}(w, z)$ denote the optimal target wages for unemployed and employed workers, then we can express them in terms of a single state variable as $w^*_{u,t}(z) = \hat{w}^*_{ut} - z - p_t$ and $w^*_{jj,t}(w, z) = \hat{w}^*_{jj,t}(w - z - p_t) - z - p_t$.

Next, we present the equilibrium conditions, and for a detailed derivation and explanation of these conditions, we refer to Blanco *et al.* (2022b).

Employed Worker's Optimality Conditions. The employed worker's decision-making involves choosing a search strategy, which includes a search intensity $s_t^*(\hat{w})$ and a submarket $\hat{w}_{jj,t}^*(\hat{w})$, and the set of states where staying in the match is optimal, referred to as the worker's continuation set $\hat{\mathcal{W}}_t^{h*}$. Given the firm's optimal continuation set $\hat{\mathcal{W}}_t^{j*}$, the worker's Hamilton-Jacobi-Bellman Variational Inequality (HJBVI) equation is given by

$$\hat{\rho}\hat{H}_t(\hat{w}) = \max\Bigg\{ \underbrace{e^{\hat{w}} - \hat{\rho}\hat{U}_t}_{\text{Income net of opportunity cost}} - \underbrace{(\hat{\gamma}+\pi_t)\frac{\partial \hat{H}_t(\hat{w})}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 \hat{H}_t(\hat{w})}{\mathrm{d}\hat{w}^2}}_{\text{Idiosyncratic productivity shocks}} - \underbrace{\delta \hat{H}_t(\hat{w})}_{\text{Exog. separation}} + \frac{\partial \hat{H}_t(\hat{w})}{\partial t}$$
$$\underbrace{+ s_t^*(\hat{w}) f(\hat{\theta}_t(\hat{w}_{jj,t}^*(\hat{w})))[\hat{H}(\hat{w}_{jj,t}^*(\hat{w})) - \hat{H}_t(\hat{w})] - \frac{\mu\left(s_t^*(\hat{w})\right)^{1+1/\phi}}{1+1/\phi}}_{\text{Change in value from on-the-job search}}$$
$$+ \underbrace{\beta^+\Delta^+\hat{H}_t(\hat{w})G^+(\Delta^+\hat{J}_t(\hat{w})) + \beta^-\Delta^-\hat{H}_t(\hat{w})G^-(\Delta^-\hat{J}_t(\hat{w}))}_{\text{Change in value from positive and negative wage changes}}, 0\Bigg\}, \quad \forall \hat{w} \in \hat{\mathcal{W}}_t^{j*}, \tag{1}$$

where $\hat{\rho} := \rho - \gamma - \frac{\sigma_z^2}{2}$, and $\hat{\rho} := \gamma + \sigma^2$. In the set of states in which the firm chooses to stay in the match, equation (2.2) shows that workers first choose whether to quit and receive a normalized value of zero or stay in the match and receive the continuation value. The first term in (2.2) represents the flow income net of the opportunity cost of employment. The second term captures the effects of idiosyncratic productivity shocks and aggregate inflation on relative wages. The third term represents the value change due to exogenous separation. The second line in (2.2) captures the event in which the worker pays a search cost to meet a new firm at rate $s_t^*(\hat{w})f(\hat{\theta}_t(\hat{w}_{jj,t}^*(\hat{w})))$ and experience a change in the value of $[\hat{H}(\hat{w}_{jj,t}^*(\hat{w})) - \hat{H}_t(\hat{w})]$. The last term accounts for the change in value that the worker experiences after an on-the-job wage change. Given a bargained relative wage $\hat{w}_{b,t}^*(\hat{w})$, the firm's and worker's non-negative gains from positive wage changes are defined as

$$\Delta^+\hat{H}_t(\hat{w}) := \hat{H}_t(\max\{\hat{w}_{b,t}^*(\hat{w}), \hat{w}\}) - \hat{H}_t(\hat{w}), \;\; \Delta^+\hat{J}_t(\hat{w}) := \hat{J}_t(\max\{\hat{w}_{b,t}^*(\hat{w}), \hat{w}\}) - \hat{J}_t(\hat{w}),$$

and the non-negative gains for wage decreases are similarly defined as

$$\Delta^-\hat{H}_t(\hat{w}) := \hat{H}_t(\min\{\hat{w}_{b,t}^*(\hat{w}), \hat{w}\}) - \hat{H}_t(\hat{w}), \;\; \Delta^-\hat{J}_t(\hat{w}) := \hat{J}_t(\min\{\hat{w}_{b,t}^*(\hat{w}), \hat{w}\}) - \hat{J}_t(\hat{w}).$$

Given the presence of renegotiation costs, renegotiations with positive and negative wage changes occur at rate $G^{+}(\Delta^{+}\hat{J}_t(\hat{w}))$ and $G^{+}(\Delta^{+}\hat{J}_t(\hat{w}))$, respectively.

While the worker's HJBVI in (2.2) characterizes the worker's value function within the firm's continuation set, the following value-matching condition characterizes the worker's value outside the firm's continuation set:

$$\hat{H}_t(\hat{w}) = 0, \quad \forall \hat{w} \notin (\hat{\mathcal{W}}_t^{j*})^c; \tag{2}$$

Equation (2.2) states that when the firm decides to lay off the worker, the normalized value becomes zero.

Finally, the worker's optimal policies are characterized by the following conditions:

$$\hat{w}^*_{jj,t}(\hat{w}) = \arg\max_{\hat{w}_{jj}} f(\hat{\theta}_t(\hat{w}_{jj}))[\hat{H}_t(\hat{w}_{jj}) - \hat{H}_t(\hat{w})], \tag{3}$$

$$s^*_t(\hat{w}) = \left(\frac{f(\hat{\theta}_t(\hat{w}^*_{jj,t}(\hat{w})))[\hat{H}_t(\hat{w}^*_{jj,t}(\hat{w})) - \hat{H}_t(\hat{w})]}{\mu} \right)^{\phi}, \tag{4}$$

$$\hat{\mathcal{W}}_t^{h*} = \left\{ \hat{w} : \hat{H}_t(\hat{w}) > 0 \; or \; e^{\hat{w}} - \hat{\rho}\hat{U}_t > 0 \right\}. \tag{5}$$

Equation (2.2) represents the submarket where the worker chooses to search for a new job and determines the corresponding wage. Equation (2.2) defines the search intensity based on the change in value resulting from on-the-job search. Lastly, Equation (2.2) characterizes the worker's continuation set, which consists of states where continuing in the match is either strictly better than quitting or a weakly dominating strategy. First, the worker stays in the match whenever the corresponding value is positive. Staying in the match is also a weakly dominating strategy if the flow relative wage is larger than the flow opportunity cost of staying in the match.[2] Note that the conditions characterizing the decision to quit can be described in terms of the more familiar value-matching condition—which requires the continuity of the value function in the entire domain—and the smooth-pasting condition—which characterizes the optimal continuation set by requiring the differentiability of the value function in the subset of the domain where the worker chooses whether to quit. That is, these conditions require that $\hat{H}_t(\cdot) \in \mathbb{C}^1(\hat{\mathcal{W}}_t^{j*}) \cap \mathbb{C}(\mathbb{R})$, where $\mathbb{C}^n(\mathcal{D})$ denotes the set of differentiable functions up to order n in the domain $\mathcal{D}$.

[2]This condition is necessary to rule out trivial equilibria that arise because of an agent's indifference when the other agent is dissolving the match (e.g., when the worker and the firm choose to dissolve the match even though flow profits and flow wages net of the flow opportunity cost are both positive). Our equilibrium refinement resolves such indifference in favor of staying in the match. See Blanco *et al.* (2022b) for further details.

Firm's Optimality Conditions. Given the worker's optimal continuation set $\hat{\mathcal{W}}_t^{h*}$, the firm's HJBVI is given by

$$\hat{\rho}\hat{J}_t(\hat{w}) = \max\Bigg\{e^{a_t} - e^{\hat{w}} - (\hat{\gamma} + \pi_t)\frac{\partial \hat{J}_t(\hat{w})}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 \hat{J}_t(\hat{w})}{\partial \hat{w}^2} - \left(\delta + s_t^*(\hat{w}) f(\hat{\theta}_t(\hat{w}^*_{jj,t}(\hat{w})))\right)\hat{J}_t(\hat{w}) + \frac{\partial \hat{J}_t(\hat{w})}{\partial t} \tag{6}$$
$$+ \underbrace{\beta^+ \int \max\{\Delta^+ \hat{J}_t(\hat{w}) - \psi, 0\}\, \mathrm{d}G^+(\psi) + \beta^- \int \max\{\Delta^- \hat{J}_t(\hat{w}) - \psi, 0\}\, \mathrm{d}G^-(\psi)}_{\text{Change in value from positive and negative wage changes}}, 0\Bigg\}, \ \forall \hat{w} \in \mathcal{W}_t^{h*}.$$

Whenever the worker chooses to stay in the match, the firm has the choice of whether to fire the worker. Condition on staying in the match, the firm receives normalized flow profits $e^{a_t} - e^{\hat{w}}$. Next, the firm's value can change either because of productivity shocks, an exogenous separation, or the worker being poached by another firm. The firm's value function is also affected by the arrival of positive and negative wage changes, which are characterized by the last term in (2.2). As explained before, the firm's value is also characterized by the value-matching and smooth-pasting conditions $\hat{J}_t(\cdot) \in \mathbb{C}^1(\hat{\mathcal{W}}_t^{h*}) \cap \mathbb{C}(\mathbb{R})$, with the value-matching condition requiring that

$$\hat{J}_t(\hat{w}) = 0, \quad \forall \hat{w} \notin (\hat{\mathcal{W}}_t^{h*})^c, \tag{7}$$

i.e., the firm's value is zero outside the worker's continuation set. Following a similar logic behind the worker's continuation set, the firm's optimal continuation set includes all relative wages for which either the firm's value function or flow profits are positive:

$$\hat{\mathcal{W}}_t^{j*} = \left\{\hat{w} : \hat{J}_t(\hat{w}) > 0 \ or \ 1 - e^{\hat{w}} > 0\right\}. \tag{8}$$

Unemployed Worker's Optimality Conditions. The unemployed worker's choice of the search strategy $\hat{w}^*_{u,t}$ is characterized by the following Hamilton-Jacobi-Bellman (HJB) equation

$$\hat{\rho}\hat{U}_t = \tilde{B} + \underbrace{f(\hat{\theta}_t(\hat{w}^*_{u,t}))\hat{H}_t(\hat{w}^*_{u,t})}_{\text{Change in value from finding a job}} + \frac{\partial \hat{U}_t}{\partial t} \tag{9}$$

and the corresponding optimality condition

$$\hat{w}^*_{u,t} = \arg\max_{\hat{w}_u} f(\hat{\theta}_t(\hat{w}_u))\hat{H}_t(\hat{w}_u). \tag{10}$$

This equation captures the trade-off faced by unemployed workers, considering the value of finding a job quickly vs. finding a job that pays more.

Free Entry Condition. The expected benefit of posting a vacancy is determined by the product of the vacancy-filling rate $q(\hat{\theta}_t(\hat{w}))$ and the firm's value of the match $\hat{J}_t(\hat{w})$. We assume that there is free entry of firms, which implies that expected profits net of the vacancy-posting cost $\tilde{K}$ must be non-positive:

$$\tilde{K} - q(\hat{\theta}_t(\hat{w}))\hat{J}_t(\hat{w}) \geq 0, \tag{11}$$

and $\hat{\theta}_t \geq 0$, with complementary slackness for all $\hat{w}$.

On-the-job Wage Renegotiation. When the worker and the firm mutually agree to renegotiate the nominal wage, the newly renegotiated wage is characterized by the Nash bargaining solution:

$$\hat{w}^*_{b,t}(\hat{w}) = \arg\max_{\hat{w}_b} \left(\hat{H}_t(\hat{w}_b) - \hat{H}_t(\hat{w})\right)^{\chi} \left(\hat{J}_t(\hat{w}_b) - \hat{J}_t(\hat{w})\right)^{1-\chi}, \tag{12}$$

subject to $\hat{H}_t(\hat{w}_b) \geq \hat{H}_t(\hat{w})$ and $\hat{J}_t(\hat{w}_b) \geq \hat{H}_t(\hat{w})$.

Equilibrium Definition We are now ready to formally define a recursive equilibrium.

Definition 1. *Given a sequence (a_t, p_t), a recursive equilibrium consists of a set of value functions $\{\hat{U}_t, \hat{H}_t(\hat{w}), \hat{J}_T(\hat{w})\}$, a market tightness-function $\hat{\theta}_t(\hat{w})$, and policy functions $\{\hat{\mathcal{W}}^{h*}_t, \hat{\mathcal{W}}^{j*}_t, w^*_{u,t}, w^*_{jj,t}(\hat{w}), s^*_t(\hat{w}), w^*_{b,t}(\hat{w})\}$ such that:*

1. *Given $\hat{U}_t$, $\hat{w}^*_{b,t}(\hat{w})$, $\hat{J}_t(\hat{w})$ and $\hat{\mathcal{W}}^{j*}_t$, the worker's value $\hat{H}_t(\hat{w})$ satisfies (2.2) and (2.2) with policies $\{\hat{w}^*_{jj,t}(\hat{w}), s^*_t(\hat{w})\}$ and the continuation set $\hat{\mathcal{W}}^{h*}_t$ given by (2.2), (2.2) and (2.2).*

2. *Given the worker's search strategy $\{s^*_t(\hat{w}), \hat{w}^*_{jj,t}(\hat{w})\}$ and continuation set $\hat{\mathcal{W}}^{h*}_t$, the firm's value $\hat{J}_t(\hat{w})$ satisfies (2.2) and (2.2) with the continuation set $\hat{\mathcal{W}}^{j*}_t$ given by (2.2).*

2. *Given $\hat{H}_t(\hat{w})$ and $\hat{\theta}_t(\hat{w})$, the value $\hat{U}_t$ satisfies (2.2) with the search policy $\{\hat{w}^*_{u,t}\}$ given by (2.2).*

3. *Given $\hat{J}_t(\hat{w})$, the market-tightness function $\hat{\theta}_t(\hat{w})$ solves the free entry condition (2.2).*

4. *Given $\hat{J}_t(\hat{w})$ and $\hat{H}_t(\hat{w})$, $\hat{w}^*_{b,t}(\hat{w})$ solves the bargaining problem (2.2).*

2.3 Equilibrium Policies

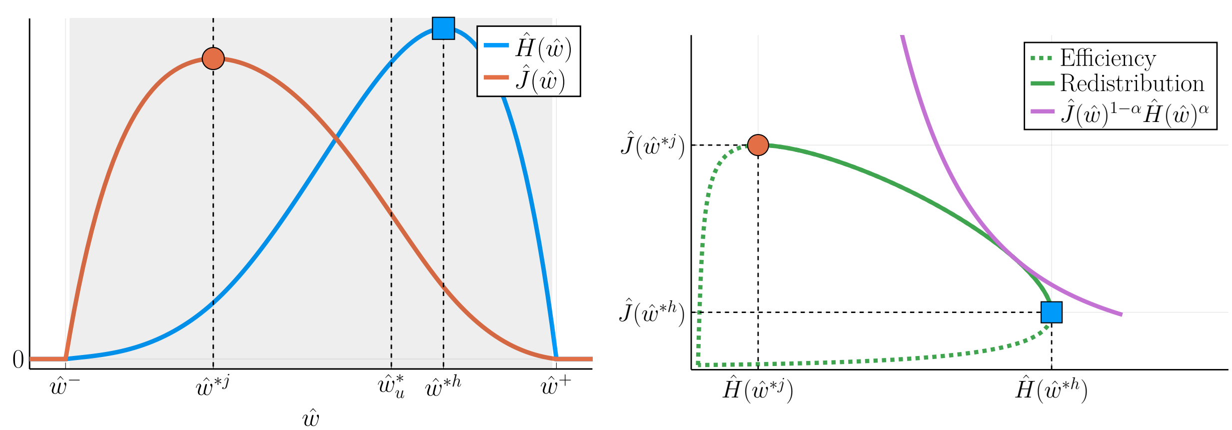
The proposed model includes three mechanisms for wage adjustment: (1) wage changes following job separations into unemployment, (2) wage renegotiation within a match, and (3) wage adjustment following job-to-job transitions. Next, we focus on the steady state and describe the economic mechanisms underlying these observable implications. For the calibration behind these figures, see Table 1.

Wage Changes Following Job Separations. In our model, there are two types of job separations: exogenous separations, occurring randomly at a rate $\delta\, \mathrm{d}t$, and endogenous separations, resulting from optimal decisions made by workers and firms. We explain the nature of endogenous separations with the help of Figure 1a, which shows the value functions of workers and firms as a function of the relative wage $\hat{w}$ (the two solid lines) and the continuation region of the match $\hat{\mathcal{W}}^{j*} \cap \hat{\mathcal{W}}^{h*} = (\hat{w}^-, \hat{w}^+)$ (the shaded area). At the beginning of the match, the worker's relative wage is given by $\hat{w}_u^*$, which then fluctuates due to productivity shocks. When the relative wage decreases below the threshold $\hat{w}^-$, the worker chooses to quit: The nominal wage and the continuation value of the match are too low relative to her productivity, so the worker quits to find a new job that pays more accordingly to her productivity.[3] Similarly, when the relative wage is above $\hat{w}^+$, the firm opts to lay off the worker. Consequently, following endogenous separations, the worker's relative wage resets back to $\hat{w}_u^*$.

FIGURE 1. WAGE RENEGOTIATION

(A) VALUE FUNCTIONS

(B) UTILITY POSSIBILITY SET



Notes: Panel A shows the value function of the worker $\hat{H}(\hat{w})$ and the firm $\hat{J}(\hat{w})$, together with the boundaries of the continuation region between $\hat{w}^-$ and $\hat{w}^+$, and the entry wage form unemployment $\hat{w}_u$. Panel B plots the combination of $\hat{H}(\hat{w})$ and the firm $\hat{J}(\hat{w})$ for all $\hat{w}$—see equation 2.3. The solid green line denotes these combinations for $\hat{w} \in (\hat{w}^{*j}, \hat{w}^{*h})$; i.e., relative wages between the relative wage that maximizes the firm's and worker's value. The dashed green line denotes these combination for $\hat{w} \notin (\hat{w}^{*j}, \hat{w}^{*h})$. Using the free-entry condition, the solid purple line denotes the trade-off behind the unemployed worker's choice of $\hat{w}_u$.

[3]Above this threshold, the worker's value first increases in the relative wage because the worker enjoys a higher utility from the higher compensation. However, the value starts decreasing at even higher relative wages because of the higher layoff risk.

To understand the determinants of the relative entry wage, observe that from the free-entry condition, we have that $\hat{\theta}(\hat{w}) = (\hat{J}(\hat{w})/\tilde{K})^{1/\alpha}$ for all $\hat{\theta} > 0$. Since $f(\hat{\theta}) = \hat{\theta}^{1-\alpha}$, worker's optimality implies that

$$\hat{w}_u^* = \arg\max_{\hat{w}_u} f(\hat{\theta}_t(\hat{w}_u))\hat{H}(\hat{w}_u) = \arg\max_{\hat{w}_u} \hat{J}(\hat{w}_u)^{\frac{1-\alpha}{\alpha}}\hat{H}(\hat{w}_u) = \arg\max_{\hat{w}_u} \hat{J}(\hat{w}_u)^{1-\alpha}\hat{H}(\hat{w}_u)^{\alpha}. \tag{13}$$

Thus, the optimal entry wage is determined by balancing the trade-off between a higher wage and a higher job-finding probability and is set in a way that resembles the Nash Bargaining solution.[4]

On-the-job Wage Renegotiation. In our framework, the relative wage of a match serves two purposes. First, it determines how the surplus is distributed between the firm and the worker. Second, it influences the efficiency of the match by affecting the probability of job separations to unemployment or a new match.

To formalize these two roles, we define the utility possibility set of the match UP given by

$$UP := \left\{(\hat{H},\hat{J}) : \exists\, \hat{w} \in (\hat{w}^-, \hat{w}^+) \textit{ such that } (\hat{H}(\hat{w}), \hat{J}(\hat{w})) = (\hat{H},\hat{J})\right\}$$

and shown it in Figure 1b. The increasing part of the utility possibility set—the dotted green line—represents the pair of worker and firm values evaluated at relative wages close to the separation thresholds $\hat{w}^-$ and $\hat{w}^+$. In these points of the state space, there exist other wages that can offer a Pareto improvement. Thus, the worker and the firm mutually agree to renegotiate the wage if the renegotiation cost is small enough. This property is illustrated in Figure 2a, which shows the bargaining hazard rate; i.e., the probability of on-the-job wage renegotiation in a small period dt. However, there is a subset of the utility possibility set—the solid green line in Figure 1b—where wage changes primarily impact the redistribution of the surplus. In these points of the state space, there is no Pareto-improving bargaining outcome, and therefore, the nominal wage remains unchanged. This requirement of mutual agreement to renegotiate the wage leads to *endogenous* wage rigidity, in addition to the rigidity resulting from the renegotiation cost.

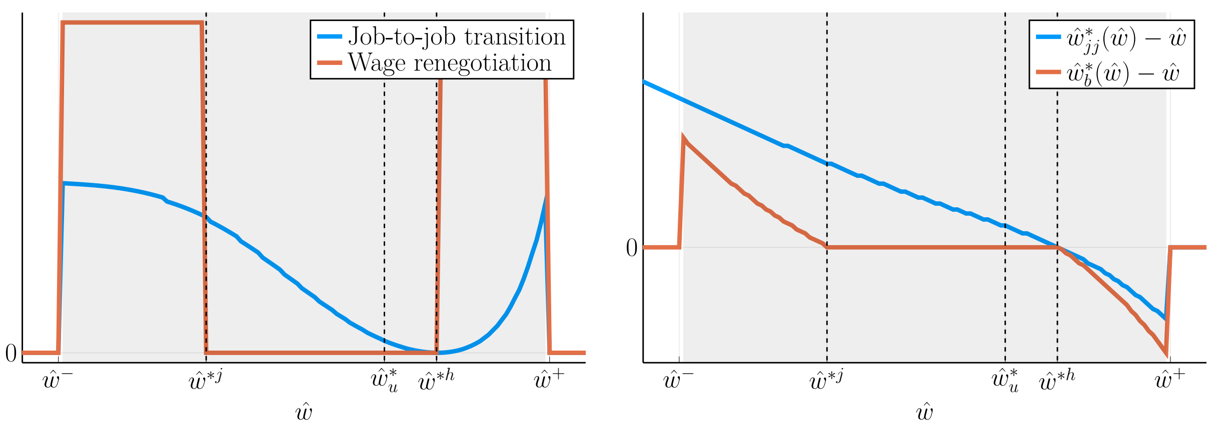
Figure 2b depicts the optimal state-contingent wage changes $\Delta_b w := \hat{w}_b^*(\hat{w}) - \hat{w}$. To understand why these are state-contingent, assume that $\chi = \alpha$. If the relative wage is close to any separation threshold, then $\hat{H}(\hat{w}) = \hat{J}(\hat{w}) \approx 0$. In this case, from equations (2.2) and (2.3), we have that $\hat{w}_b^*(\hat{w}) \approx \hat{w}_u^*$; i.e., the bargained wage approximates the entry wage, as the opportunity costs for both the worker and the firm are equal to their respective values when unmatched. On the other hand, if the relative wage falls between

[4] This fact does not imply that $\hat{H}(\hat{w}_u^*) = \alpha\hat{S}(\hat{w}_u^*)$—where $\hat{S}(\hat{w}) := \hat{H}(\hat{w}) + \hat{J}(\hat{w})$—since values are *non-linear* functions of relative wages (see Blanco *et al.*, 2022b).

$\hat{w}^{*j} = \arg\max_{\hat{w}} \hat{J}(\hat{w})$ and $\hat{w}^{*h} = \arg\max_{\hat{w}} \hat{H}(\hat{w})$, then $\Delta_b w = 0$ since there are no Pareto-improving outcomes in this range.

FIGURE 2. WAGE ADJUSTMENT WITHIN AND ACROSS JOBS

(A) HAZARD RATES (B) WAGE CHANGES



Notes: Panel A plots the hazard rate for job-to-job transitions (solid blue line) and on-the-job wage renegotiation (solid orange line). Panel B shows the target wage the employed worker is searching for (relative to the current wage) and the outcome of the wage renegotiation process (relative to the current wage).

Wage Adjustment Following Job-to-job Transitions. The worker's on-the-job search policy consists of a target wage $\hat{w}^*_{jj}(\hat{w})$ and a job-to-job hazard rate given by $s^*(\hat{w}) f(\hat{w}^*_{jj}(\hat{w}))$. The target wage is determined by equation (2.2), which is similar to equation (2.2) for the entry wage but with a higher opportunity cost for the worker. Since the opportunity cost depends on the current relative wage, so does the target wage and the wage adjustment following a job-to-job transition. This implies that wage increases follow job switches for sufficiently low relative wages—i.e., the model features a job ladder—and wage decreases for sufficiently high relative wages. To see this, assume $\sigma = \beta^- = \beta^+ = 0$. We can define the sequence of relative wages following the k-th job-to-job transition as $\hat{w}^k = \hat{w}^*_{jj}(\hat{w}^{k-1})$ with $\hat{w}^0 = \hat{w}^*_u$. Then, the wage change following the k-th transition is $\Delta_{jj} w^k = \hat{w}^k - \hat{w}^{k-1} > 0$ for a low relative wage $\hat{w}^{k-1}$. Moreover, the hazard rate decreases whenever the current relative wage is closer to $\hat{w}^{*h}$, which represents the relative wage that maximizes the worker's value. This happens because the marginal gain from on-the-job search becomes smaller, and search is costly.

It is worth noting that the interaction between limited commitment and on-the-job search leads to wage decreases as the optimal choice for the worker following a job-to-job transition. Consider a situation where a worker experiences a sufficiently negative productivity shock. Due to wage stickiness, the wage does not reflect the worker's lower productivity, giving the firm an incentive to lay off the worker. To avoid unemployment, the worker searches for a new wage that pays a lower wage than the current match. Consequently, the model can generate job-to-job transitions due to "unsatisfactory pay" (relative wages below $\hat{w}^{*h}$) or "fear of losing the job" (relative wages above $\hat{w}^{*h}$), two phenomena that have been empirically documented by Fujita (2010).

3 Model Application and Parametrization

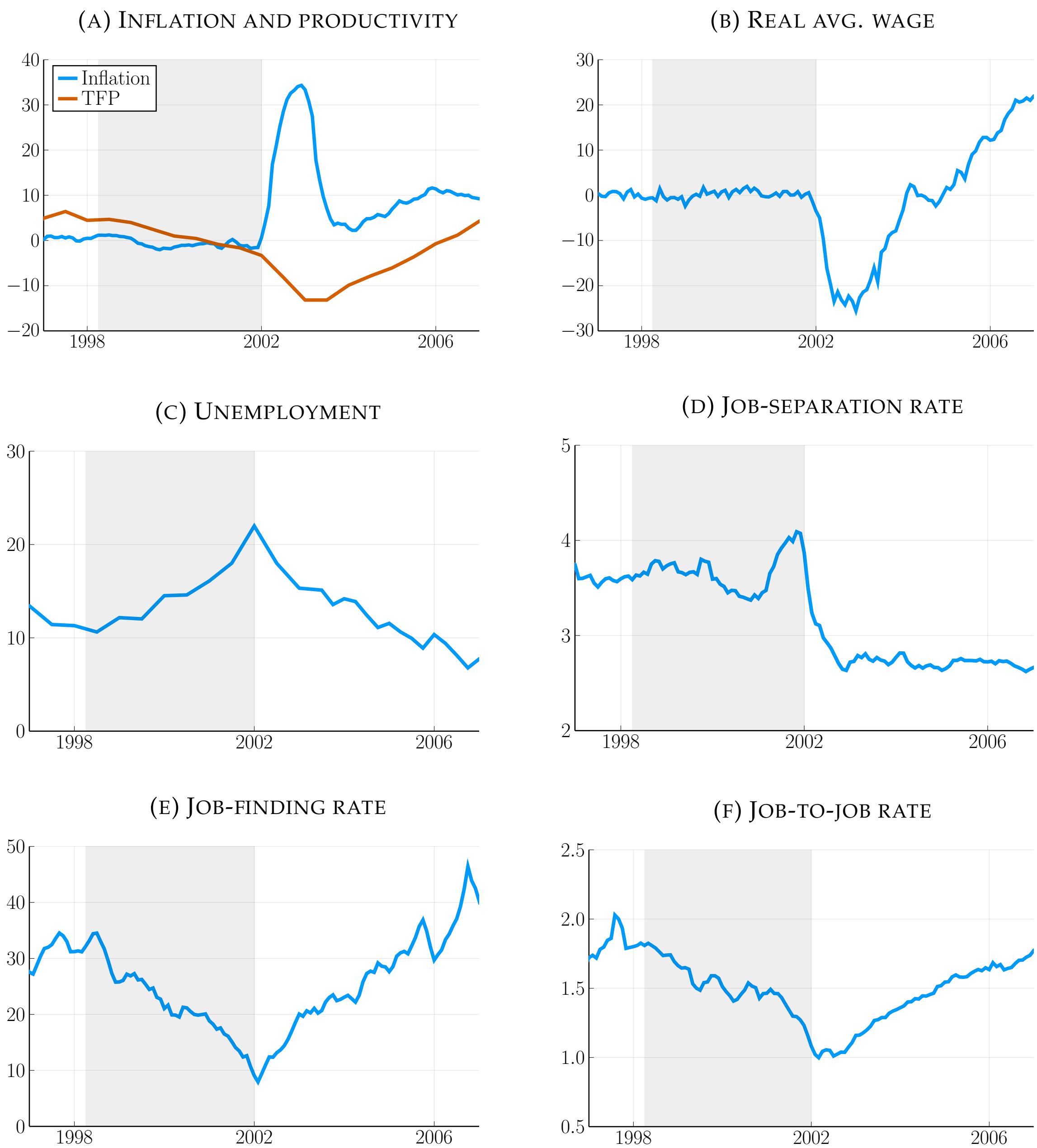
We now proceed to analyze the quantitative predictions of the model. We first describe the "want operator"—i.e., the set of facts we want the model helps us understand. Section 3.1 presents the labor market dynamics around the Argentina 2001 crisis, culminating in a significant inflation increase. In Section 3.2, we then describe the data and the moments we use to discipline the model and the estimation procedure.

3.1 Application: Argentina 2001 Crisis

Our study aims to examine the impact of inflation on labor market dynamics. We believe the 2001 Argentina crisis is an ideal case study due to the significant inflation surge experienced during a recession. This section provides an overview of the crisis and the associated labor market dynamics. For a more comprehensive description of this event and the data sources used to construct labor market variables, we refer readers to Blanco *et al.* (2022c). We highlight that aggregate dynamics during this episode resemble cross-country aggregate dynamics during large devaluations.

During the analyzed period, Argentina's macroeconomic environment was characterized by a history of high and volatile inflation. In an attempt to address this issue, the country implemented a currency board regime in April 1991, pegging its currency to the U.S. dollar. This policy successfully brought inflation under control, providing nominal stability and fostering rapid economic growth. However, the economy encountered significant challenges in 1998 when it entered a recession following the Russian crisis, which led to reduced capital flows and increased sovereign spreads. Additionally, the devaluation of the Brazilian currency (Argentina's main trading partner), the global appreciation of the U.S. dollar, and the substantial decline in commodity prices further compounded the economic difficulties. As a result, the fiscal deficit deteriorated, public debt increased, and an austerity plan was implemented, exacerbating the recession. Consequently, the country experienced significant capital flight, a run on the local banking system, and ultimately defaulted on its external debt, leading to a freeze on deposits and the abandonment of the exchange-rate peg.

FIGURE 3. LABOR MARKET DYNAMICS OVER THE 2001 ARGENTINA CRISIS



Notes: The figure shows aggregate time series between January 1997 and December 2006. Panel A plots a measure of aggregate TFP and the year-over-year inflation rate. The TFP measure is constructed as output per worker (the ratio between real GDP and total employment from the national household survey EPH); the series is normalized to its value in the first semester of 2001 and expressed in log points $\times 100$. The shaded area denotes the recession period. Panel B shows the average real labor income from SIPA. Panel C plots the unemployment rate from EPH. Panels D, E, and F show the job-separation rate to non-employment, the job-finding rate, and the job-to-job rate, which are constructed by combining data from EPH and SIPA, respectively. We define a job-to-job transition as a worker's change of employers with an intervening unemployment spell of at most one month.

Figure 3 presents the aggregate time series surrounding this episode. Panel 3a illustrates the evolution of aggregate inflation and output per worker, which serves as a simple proxy for total factor productivity (TFP). To analyze the labor market response to the January 2002 inflation surge, we utilize a combination of microdata from the administrative social security records known as the "Sistema Integrado Previsional Argentino" (SIPA) and the national household survey referred to as the "Encuesta Permanente de Hogares" (EPH). Panels 3b to 3f depict the changes in average real labor income, the unemployment rate, the job-separation rate, the job-finding rate, and the job-to-job rate over time.

The January 2002 devaluation allows for the analysis of the labor market response to a policy change that generated a large and unexpected increase in inflation. Prior to the devaluation, Argentina experienced a period of low and stable inflation alongside declining TFP. However, following the devaluation, year-over-year inflation surged to approximately 35%, while aggregate TFP further declined by an additional 10 percentage points and remained below the 2001 level for an extended period. Interestingly, average real labor income remained relatively stable in the five years leading up to the devaluation, despite a significant cumulative decrease in TFP. However, after the devaluation, there was a sharp decline of 25% in average labor income, which can be attributed to the delayed response in the frequency of job wage changes (see Blanco *et al.*, 2022a). Concurrently, the unemployment rate, which had been steadily increasing prior to the devaluation, experienced a persistent and rapid decline, aligning with the increase in inflation and the drop in real labor income. This labor market recovery is evident across all labor flows. The spike in the job-separation rate observed the year before the inflation surge was quickly reversed in the months that followed. Furthermore, the downward trends in job-finding and job-to-job separations halted with the increase in inflation and gradually improved thereafter.

These facts surrounding the inflation surge present a challenge to existing models of frictional labor markets. Although these models would have no problems replicating the labor market dynamics before the increase in inflation, they would face difficulty in explaining, for example, the rapid and significant decline in the job-separation rate into unemployment amidst declining TFP. As we show below, through the lens of our model, these facts are not a puzzle; the key is to have a realistic model for the adjustment of nominal wages and endogenous separations.

3.2 Calibration Strategy

A period in the model corresponds to a month. We set the discount factor ρ to an annualized value of 4%. Given the lack of data on vacancies in Argentina, we set the elasticity of the matching function with respect to effective units of searchers α to 0.72 as in Shimer (2005b). We assume a parametric distribution for the renegotiation cost given by $G^{-}(\psi) = G^{+}(\psi) = 1$. Table 1 reports the model parameters together with their respective targets.

TABLE 1. PARAMETER VALUES AND IMPLICIT TARGETS IN SMM

Parameter	Description	Value	Target
		Panel A: External Calibration	
ρ	Discount factor	0.04/12	4% annual real interest rate
α	Elasticity matching function	0.72	Shimer (2005b)
		Panel B: Interal Calibration	
$\tilde{K}$	Vacancy posting cost	31.00	Job-finding rate
$\tilde{B}$	Flow value of unemployment	0.19	Avg. income unemployed / employed
γ	Drift worker's productivity shock	0.00	Avg. wage change during EUE transitions
σ	St. de. worker's productivity shock	0.05	St. dev. wage change during EUE transitions
μ	Level of search cost	3.26	Job-to-job transition rate
ϕ	Elasticity of search cost	0.99	Distribution wage changes during EE transitions
χ	Worker's bargaining power	0.33	Distribution wage changes on-the-job
β^{+}	Arrival free positive wage changes	0.51	Freq. positive wage changes on-the-job
β^{-}	Arrival free negative wage changes	0.10	Freq. negative wage changes on-the-job

Notes: The table presents the parameter values assigned to the model.

We estimate the remaining 9 parameters to match moments of the wage-change distribution and labor flows in Argentina from 1996 to 2000, which captures a low inflation environment. For this, we use the steady-state solution of our model to compute the same statistics in the model as in the data. We then chose the parameter set $\mathcal{P} = \{\tilde{K}, \tilde{B}, \gamma, \sigma, \mu, \phi, \chi, \beta^{+}, \beta^{-}\}$ that minimizes the objective $(m_m(\mathcal{P}) - m_d)' W (m_m(\mathcal{P}) - m_d)$, where m_m and m_d are vectors of model-simulated moments and data moments, respectively, and W is a diagonal matrix. Tables 2 and 3 describe the empirical targets used to discipline the parameters in our model. Next, we describe these data and the steps followed to compute the moments in m_d.

Moment construction in the data. We use administrative employer-employee-matched monthly data from Argentina's national social security system. The main variable we use is workers' total pre-tax nominal monthly compensation in the formal sector, which includes all forms of compensation (i.e., base wage, bonuses, etc.). From now on, we refer to this variable as the worker's nominal wage. For more details about the data, see Blanco *et al.* (2022c).

The distribution of wage changes is critical to discipline the response of aggregate wages and employment to aggregate shocks. For this reason, we apply several filters to render the data compatible with the model. First, we perform the following sample selection procedure. We restrict our sample to workers aged between 25 and 55 in the private sector to avoid issues related to early retirement or non-market wage determination. We additionally eliminate workers earning less than half of the monthly minimum wage and winsorize nominal wages at the top 99.999th percentile.

Second, in the spirit of Barattieri *et al.* (2014), we apply a filter to nominal wages to recover workers' "regular" wages. Although our administrative data may contain negligible measurement error, our model abstracts from many sources of transitory deviations from a modal or permanent wage. In the data, transitory deviations can arise from two main sources. First, small transitory deviations around a permanent wage can result from the intensive margin of labor supply or worker's commission. Second, significant transitory deviations can result from seasonal factors, such as end-of-the-year bonuses, vacation payments, or the payment of the 13th salary. Given that our model abstracts from many of these features, we eliminate them from the data by applying the following filter.

We construct a measure of the regular wage following the Break Test methodology developed in Blanco *et al.* (2022b). In a nutshell, the logic behind this methodology is to split a nominal wage series into two continuous subsamples and perform a statistical test of whether those subsamples were drawn from the same distribution using the Kolmogorov-Smirnov statistic. The methodology will identify changes in the regular wage series whenever differences between observed wage series before and after a potential break are sufficiently large.[5] To illustrate the importance of removing transitory wage changes, using the same data Blanco *et al.* (2022b) show that the *monthly* frequency of wage changes based on the unfiltered data is 70%, while the frequency of regular wage changes is 13%, which is much aligned with the estimates in Barattieri *et al.* (2014) and Grigsby *et al.* (2021).

Table 2 describes the wage change distribution arising from on-the-job bargaining (columns denoted by "Δw Bargaining"), the wage change distribution from job-to-job transitions ("Δw EE"), and the wage change distribution following a job-separation into unemployment ("Δw EUE"). For each transition, we remove outliers by dropping observations below the 1% and above the 99% percentiles. In addition, our model abstracts from wage differences arising from firm heterogeneity; in the model, productivity differences are only driven by workers' idiosyncratic shocks. Therefore, we filter the data on wage changes across firms to remove the proportion of wage changes arising from firms' fixed heterogeneity. The contribution of firm heterogeneity to wage dispersion has been previously documented by Card *et al.* (2016); Schmutte (2015); Jinkins and Morin (2018), among others. Based on the methodology developed by

[5]This methodology relies on one parameter $\mathcal{K}$, which defines the critical values for the test to reject the null hypothesis of no break in the series. Since there are no standardized critical values, Blanco *et al.* (2022b) estimates $\mathcal{K}$ combining a cross-validation exercise with a statistical model of wage changes, in which wages are negotiated whenever the relative wage hits a lower or upper (S,s) band—a model observationally close to the one presented in Section 2.

TABLE 2. TARGETED MOMENTS: WAGE-CHANGE DISTRIBUTIONS

	Δw Bargaining		Δw EE		Δw EUE	
	Data	Model	Data	Model	Data	Model
Mean	0.01	-0.00	0.04	0.09	-0.04	-0.06
Standard deviation	0.13	0.07	0.19	0.16	0.30	0.18
Kurtosis	3.73	5.32	4.53	2.99	3.34	2.80
10th Percentile	-0.15	-0.11	-0.17	-0.19	-0.40	-0.28
25th Percentile	-0.07	-0.03	-0.05	0.02	-0.20	-0.21
50th Percentile	0.00	0.02	0.01	0.13	-0.02	-0.07
75th Percentile	0.08	0.04	0.12	0.20	0.12	0.06
90th Percentile	0.16	0.06	0.29	0.25	0.31	0.18

Notes: The table presents selected moments of the wage-change distribution in Argentina between 1996 and 2000 used in the estimation. The first two columns report moments of wage changes within the same job in the data and the model, respectively. The following two columns report moments of wage changes from job-to-job transitions in the data and the model, respectively. The last two columns report moments of wage changes following job separations into unemployment in the data and the model, respectively.

Abowd *et al.* (1999), the main finding is that firm fixed effects account for approximately 45% of the wage level dispersion and 17% of the dispersion of wage growth of workers that switch jobs. Thus, we re-scale the distribution of wage changes across jobs by 0.6 to bridge the gap between the model and the data. Notice that this rescaling does not affect the higher-order moments of the wage-change distributions (i.e., skewness and kurtosis) and only affects the dispersion of the wage changes. As expected, wage growth following EE transitions is positive, while average growth during EUE transitions is negative. In addition, wage changes on-the-job are the least dispersed, followed by the dispersion of EE wage changes and EUE wage changes, which exhibits the largest amount of dispersion.

Table 3 reports the remaining moments used to calibrate the model. The computation of the monthly E-to-U separation and job-to-job finding rates is standard. The separation rate is slightly higher than the average separation rate of 0.034 in the U.S. (see Shimer, 2005b), and the job-to-job transition rate is half than that in the U.S. (see Nagypál, 2007; Fujita *et al.*, 2020). Two issues arise when computing the remaining U-to-E transition rate: (i) the administrative data do not contain information on the unemployed, and (ii) in the household survey we do not observe whether an unemployed worker searches for jobs in the formal or informal sector. We construct the monthly U-to-E transition rate as the product between the monthly entry rate in the formal sector UE_t/E_{t-1} and the aggregate employment-to-unemployment ratio E_{t-1}/U_{t-1}. Next, using data from the national household survey, we measure the replacement ratio by computing the ratio of a worker's income during unemployment and employment and taking the average across workers transitioning from unemployment to formal employment and vice versa. Finally, we compute the frequency of wage changes as the share of non-zero regular wage changes within the same job.

TABLE 3. ADDITIONAL TARGETED MOMENTS

Moment	Data	Model
U-to-E transition rate	0.280	0.278
Job-to-job transition rate	0.013	0.013
E-to-U transition rate	0.039	0.036
Avg. replacement ratio	0.203	0.208
Freq. of wage changes	0.086	0.067

Notes: The table presents Argentina's average labor market flows, replacement ratio, and frequency of on-the-job wage changes between 1996 and 2000.

Internal Calibration. Next, we discuss how each parameter in $\mathcal{P}$ is informed by specific sets of moments, despite being jointly calibrated. First, the value of home production $\tilde{B}$ is directly identified by the replacement ratio. Given the value of home production, the cost of posting vacancies determines the job-finding rate. We set the parameters of the idiosyncratic productivity process γ and σ to match the distribution of wage changes after a EUE transition, particularly the mean and the standard deviation. To understand this choice, assume there is no on-the-job search and no on-the-job wage renegotiation. Then, following a separation, wage changes are informative of the worker's cumulative productivity shocks experienced between the starting date of two consecutive jobs. Thus, as shown by Blanco *et al.* (2022b), γ and σ are proportional to the average and the variance of wage changes across jobs.[6]

Once these parameters are determined, the remaining parameters are identified by the frequency of wage changes, average E-to-E and E-to-U transition rates, and moments of the wage change distribution within jobs and following an E-to-E transition. The parameters μ and ϕ of the search cost function are identified by the average job-to-job transition rate and the kurtosis of the distribution of wage changes across jobs. Intuitively, μ determines the level of the job-to-job hazard rate, while ϕ determines its slope. With a given slope, a lower search cost leads to a higher job-to-job transition rate. The slope of the hazard rate is governed by ϕ. For a given change in the expected benefit of on-the-job search, a higher value of ϕ implies a greater change in search intensity by workers. When $\phi = 0$, search effort becomes state independent, and the kurtosis of the wage change distribution after E-to-E transitions should be close

[6]The proportionality factor is given by the average time elapsed between the starting date of two consecutive jobs during a EUE transition. This moment is mechanically matched when calibrating the average separation and job-finding rates.

to 6 (i.e., similar to that of the Laplace distribution). As $\phi \to \infty$, workers make E-to-E transitions almost instantaneously, resulting in a kurtosis of the wage change distribution of 1.

Finally, the hazard rates of on-the-job wage renegotiations and the worker's bargaining power, (β^-, β^+, χ), are determined by the probability of wage increases and decreases on-the-job, along with the average wage change. It is worth noting that although the calibrated hazard rates of wage increases and decreases are set at 0.5 and 0.1, respectively, the actual probability of wage adjustments is significantly lower due to the model's non-negligible degree of endogenous wage rigidity. This is because the relative wage $\hat{w}$ for many matches falls between $\hat{w}^{*j}$ and $\hat{w}^{*h}$, a region where wage renegotiations mainly impact the redistribution of match surplus rather than its efficiency. As a consequence of the large asymmetries between the hazard rates for positive and negative wage changes, the model predicts that layoffs occur much more frequently than quits, with layoffs accounting for 34% of total separations while quits comprise only 4.5%. The much higher prevalence of layoffs is consistent with the empirical evidence for the US (see Elsby *et al.*, 2010).

As Tables 2 and 3 show, the model can match the targeted moments well.

4 Quantitative Analysis

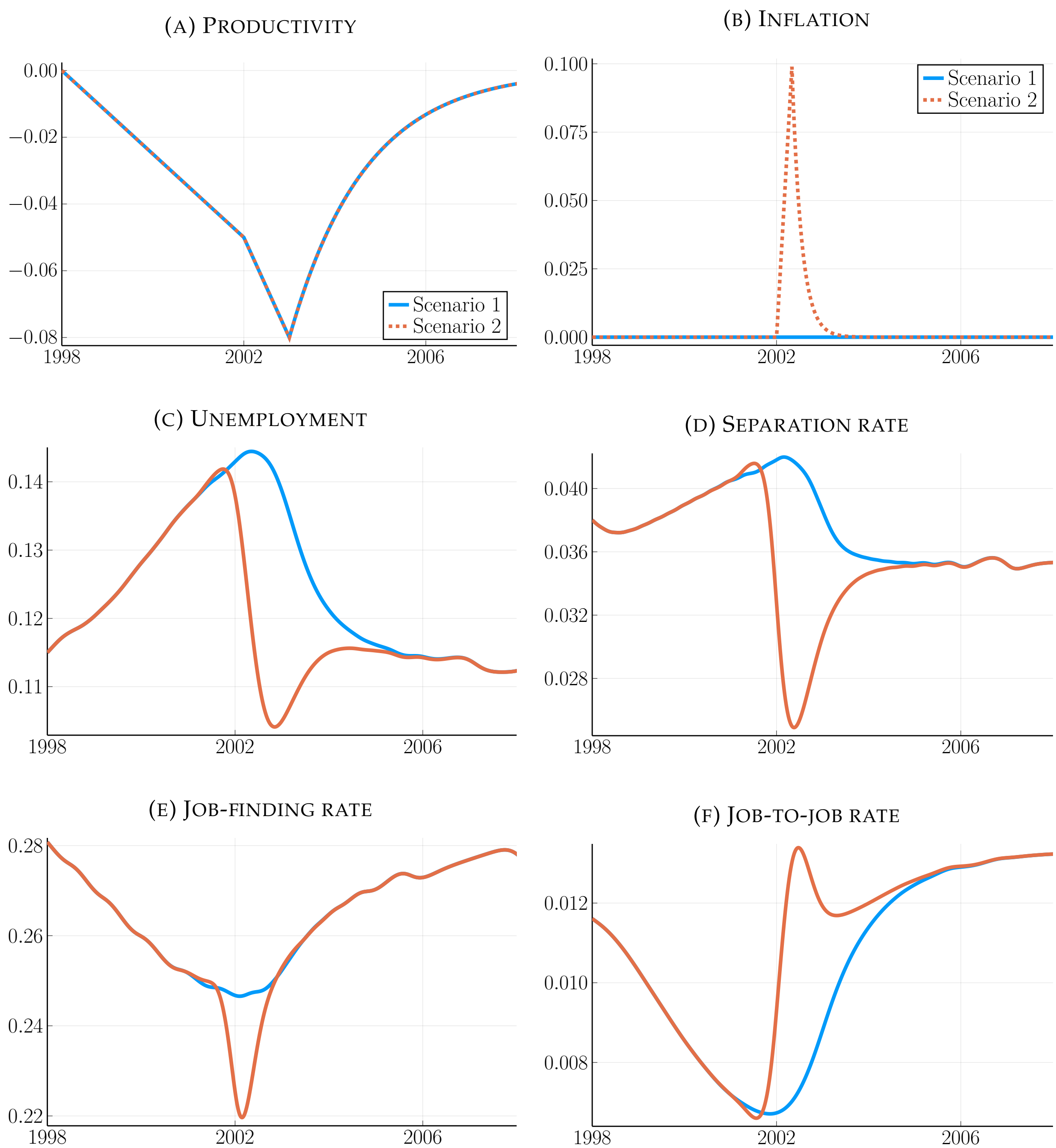
In this section, we aim to examine the quantitative predictions of our model regarding the impact of inflation on labor market dynamics. To achieve this, we simulate a series of aggregate shocks that closely mimic the 2001 Argentine recession. Our analysis considers two distinct scenarios.

In the first scenario, we assume that starting from 1998, the economy encounters a sequence of adverse productivity shocks. These shocks are initially at a rate of 1.25% per year and intensify to 3% per year at the beginning of 2002. This heightened level of shocks persists until early 2003, at which point the aggregate productivity reverts to its steady-state value (Figure 4a), with a half-life of 14 months.

The second scenario replicates the unexpected sequence of productivity shocks experienced in 1998, similar to the first scenario. However, in addition to these shocks, agents in the economy also face an unanticipated shock to inflation in early 2002. This shock increases monthly inflation linearly from 0% to a maximum of 10% over a period of four months, mirroring the actual data. Subsequently, inflation returns to 0% with a half-life of 2 months (dashed line in Figure 4b). By investigating these two scenarios, we can evaluate the differential effects of inflation when compared to the isolated productivity shocks.

Figure 4 illustrates the dynamic changes in labor flows triggered by the aggregate shocks. In the first scenario, the lower aggregate productivity lowers firms' flow profits and values. Consequently, the lower firm value induces a rise in the job-separation rate (4d), primarily driven by an increase in the layoff rate

FIGURE 4. LABOR FLOWS DYNAMICS AFTER AGGREGATE SHOCKS



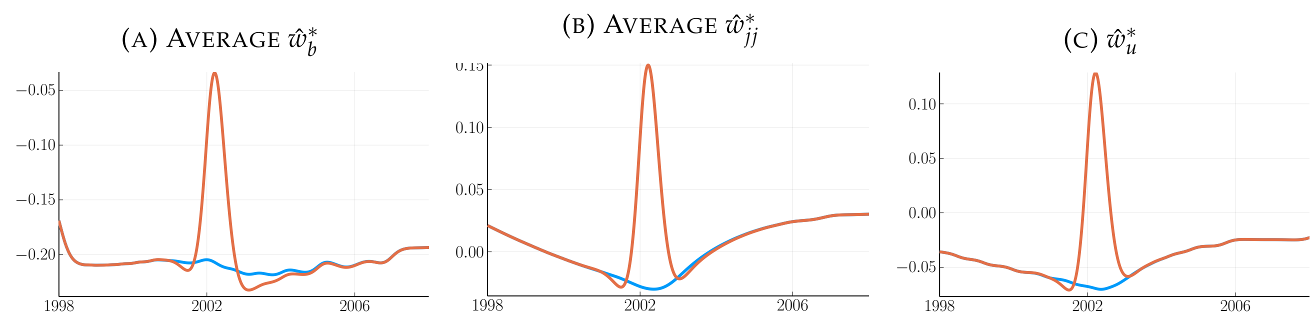
(see below). Additionally, the lower firm value also reduces the job-finding rate of the unemployed (4e). These persistent flow adjustments elevate the unemployment rate beyond the steady state level by more than 2 percentage points. Furthermore, the job-to-job transition declines both directly due to reduced vacancy postings by firms and indirectly due to diminished worker incentives for on-the-job search (4f).

In the second scenario, the unanticipated inflation shock exerts a substantial stabilizing effect on the unemployment rate, initiating a recovery despite the ongoing decline in aggregate productivity. This outcome can be attributed to a significant decrease in the job-separation rate (the job-finding rate experiences a transitory decline during the high-inflation period, as we explain below). Additionally, the inflation shock reduces real wages for employed workers, prompting a notable increase in their on-the-job search intensity.

To explain the dynamics of these flows, it is crucial to analyze the wage dynamics. Figure 5 shows the dynamics of the average wage of bargained contracts $\hat{w}^*_b$, the average wage of new hires $\hat{w}^*_{jj}$, and the average wage of unemployed workers $\hat{w}^*_u$. In response to the lower productivity, workers and firms engage in downward wage renegotiations. The willingness of workers to accept wage cuts stems from the productivity shock reducing the layoff threshold ($\hat{w}^+$) (6b). Consequently, workers agree to lower wages to avoid separations into unemployment. However, the decline in wages remains small relative to the magnitude of the shock, and aggregate separations decrease nonetheless. In response to the shock, both employed and unemployed workers actively search for jobs offering lower wages relative to the steady state, thereby preventing a further decline in the job-finding rate.

In light of the inflation shocks, workers try to secure real wage increases. A portion of employed workers successfully renegotiate for higher real wages (Figure 5a) and, due to increased on-the-job search intensity, firms are inclined to grant such increases to retain their workforce and deter employees from seeking alternative jobs. Another fraction of employed workers secures new jobs with higher real wages before renegotiating with their current employers (Figure 5b). Simultaneously, the higher inflation rate prompts unemployed workers to search for jobs offering a higher real wage, consequently reducing their effective job-finding rate.

FIGURE 5. WAGE DYNAMICS AFTER AGGREGATE SHOCKS



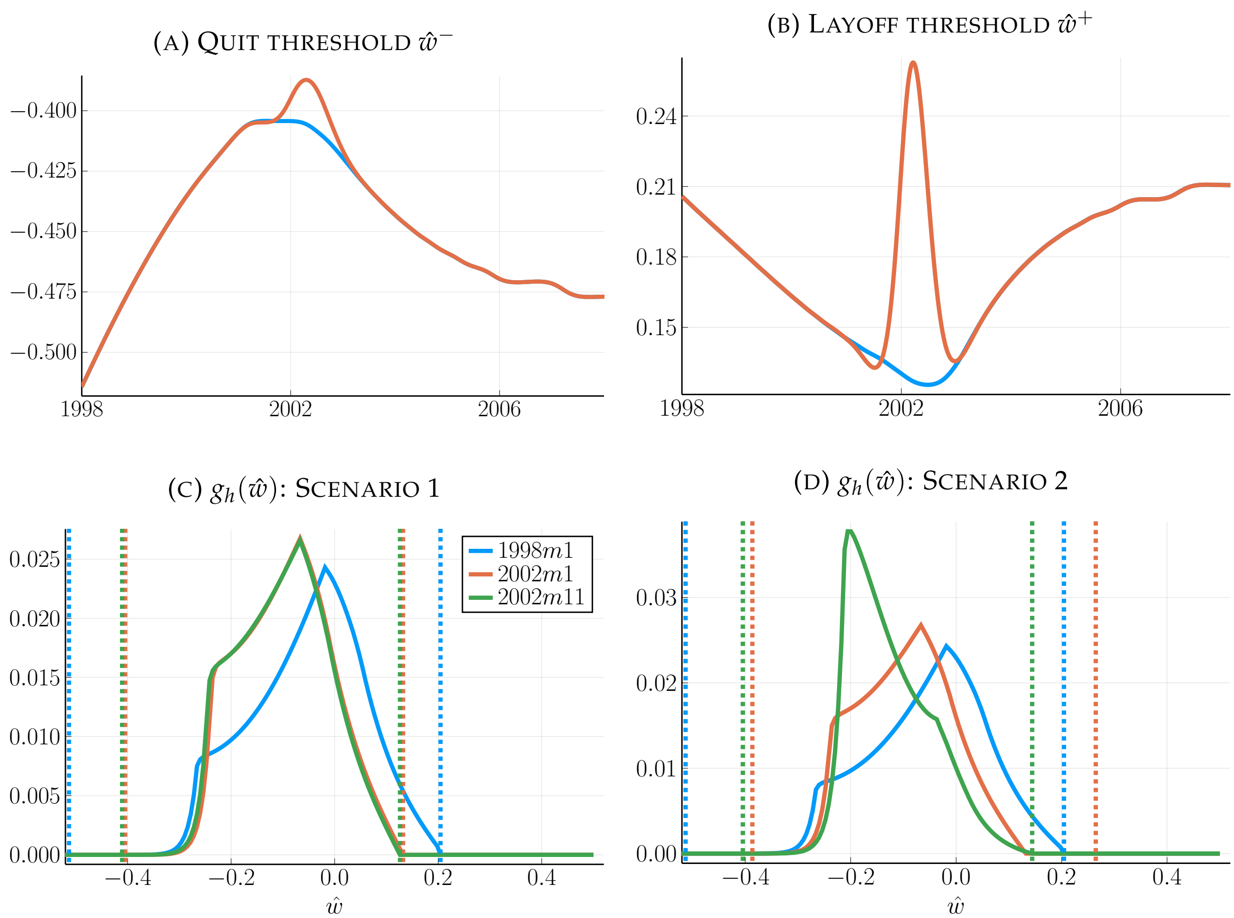
Finally, we delve into the determinants of the job-separation rate, which, in our model, serves as the primary driver of the unemployment rate. Specifically, we focus on the dynamics of its endogenous components: quits and layoffs. Figure 6 depicts the quits and layoff thresholds $\hat{w}^-$ and $\hat{w}^+$ alongside the evolution of the distribution of the real wage-to-productivity ratio $\hat{w}$ in each scenario. In the first scenario, the lower aggregate productivty leads to an increase in the quits threshold since the constant real value of home production becomes more attractive. Simultaneously, the layoff threshold decreases, indicating firms' reduced willingness to sustain matches with high $\hat{w}$. However, as depicted in Figure 6c, the impact of the first effect on the quit rate remains small, given the already low steady-state quit rate. Conversely, the second effect is more significant, emerging as the primary driver of the increased job-separation rate. This is primarily due to the persistent and significant concentration of workers near the declining layoff threshold since the beginning of the shock.

In contrast, the second scenario demonstrates that the inflation shock marginally increases the quits threshold while considerably reducing the layoff threshold. Furthermore, the inflation shock diminishes the real wage-to-productivity ratio for employed workers, shifting the distribution to the left and reducing the concentration of workers near the layoff threshold. Consequently, the job-separation rate experiences a significant decrease, enabling the unemployment rate to begin its recovery despite the ongoing decline in aggregate productivity.

5 Conclusion

This paper contributes to the analysis of wage rigidity and its implications for labor market dynamics. By developing a frictional labor market model with directed and on-the-job search, incorporating idiosyncratic worker productivity shocks, two-sided limited commitment, and wage rigidities, we provide a comprehensive framework to understand wage adjustments within and across jobs. Our model captures the state-dependent nature of wage changes, where the current wage determines the probability and magnitude of future adjustments. Grounding our analysis in microdata on wage adjustment, we calibrate the model using administrative employer-employee-match monthly labor income data from Argentina, enabling us to accurately examine the dynamics of real wages and employment. Furthermore, we highlight the role of inflation in alleviating the negative consequences of wage rigidity, as demonstrated through empirical evidence and supported by our model's ability to replicate the dynamics observed during the 2001 Argentinean recession and the subsequent policy changes. Overall, this research enhances our understanding of the complex relationship between wage rigidity, labor market frictions, and aggregate fluctuations, providing valuable insights for the design of effective fiscal, monetary, and social insurance policies.

FIGURE 6. SEPARATIONS DYNAMICS AFTER AGGREGATE SHOCKS



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How Does Inflation "Grease the Wheels" in a Frictional Labor Market?

Online Appendix—Not for Publication

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A Numerical Appendix

A.1 Solving the Steady State

Summary of Equations: Define the transformed drift $\hat{\gamma} := \gamma + \sigma^2$ and the transformed discount factor $\hat{\rho} := \rho - \gamma - \sigma^2/2$. The steady-state equilibrium satisfies the following equations:

1. Free entry condition for $\hat{\theta}(\hat{w})$:

$$\tilde{K} = \hat{\theta}(\hat{w})^{-\alpha}\hat{J}(\hat{w}).$$

2. Normalized value of unemployment:

$$\hat{\rho}\hat{U} = \tilde{B} + \hat{\theta}(\hat{w}_u^*)^{1-\alpha}\hat{H}(\hat{w}_u^*),$$

where $\hat{w}_u^* = \arg\max_{\hat{w}_u} \hat{\theta}(\hat{w}_u)^{1-\alpha}\hat{H}(\hat{w}_u)$.

3. Normalized value function of an employed worker:

$$\begin{aligned}\hat{\rho}\hat{H}(\hat{w}) = \max\Big\{0\,,\, & e^{\hat{w}} - \hat{\rho}\hat{U} - \hat{\gamma}\frac{\partial \hat{H}(\hat{w})}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 \hat{H}(\hat{w})}{\partial \hat{w}^2} - \delta\hat{H}(\hat{w}) \\ & + \beta^+\Delta^+\hat{H}(\hat{w})G^+(\Delta^+\hat{J}(\hat{w})) + \beta^-\Delta^-\hat{H}(\hat{w})G^-(\Delta^-\hat{J}(\hat{w})) \\ & + s^*(\hat{w})f(\hat{\theta}(\hat{w}_{jj}^*(\hat{w})))[\hat{H}(\hat{w}_{jj}^*(\hat{w})) - \hat{H}(\hat{w})] - \frac{\mu\,(s^*(\hat{w}))^{1+1/\phi}}{1+1/\phi}\Big\},\ \forall \hat{w} \in \hat{\mathcal{W}}^{j*}\end{aligned}$$

and $\hat{H}(\hat{w}) = 0 \quad \forall \notin \hat{\mathcal{W}}^{j*}$, where $s^*(\hat{w}) = \left(\frac{f(\hat{\theta}(\hat{w}_{jj}^*(\hat{w})))[\hat{H}(\hat{w}_{jj}^*(\hat{w}))-\hat{H}(\hat{w})]}{\mu}\right)^{\phi}$ and $\hat{w}_{jj}^*(\hat{w}) = \arg\max_{w_{jj}} f(\hat{\theta}(\hat{w}_{jj}))[\hat{H}(\hat{w}_{jj}) - \hat{H}(\hat{w})]$.

4. Normalized value function of a filled vacancy:

$$\begin{aligned}\hat{\rho}\hat{J}(\hat{w}) = \max\Big\{0\,,\, & 1 - e^{\hat{w}} - \hat{\gamma}\frac{\partial \hat{J}(\hat{w})}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 \hat{J}(\hat{w})}{\partial \hat{w}^2} - \delta\hat{J}(\hat{w}) - s^*(\hat{w})f(\hat{\theta}(\hat{w}_{jj}^*(\hat{w})))\hat{J}(\hat{w}) \\ & + \beta^+\int \max\{\Delta^+\hat{J}(\hat{w}) - \psi, 0\}\,\mathrm{d}G^+(\psi) + \beta^-\int \max\{\Delta^-\hat{J}(\hat{w}) - \psi, 0\}\,\mathrm{d}G^-(\psi)\Big\},\ \forall \hat{w} \in \hat{\mathcal{W}}^{h*}\end{aligned}$$

and $\hat{J}(\hat{w}) = 0 \quad \forall \hat{w} \notin \hat{\mathcal{W}}^{h*}$.

5. Optimal bargaining: $\Delta^+\hat{J}(\hat{w}) = \hat{J}(w_+ - z) - \hat{J}(w - z)$, $\Delta^-\hat{J}(\hat{w}) = \hat{J}(w_- - z) - \hat{J}(w - z)$,$\Delta^+\hat{H}(\hat{w}) = \hat{H}(w_+ - z) - \hat{H}(w - z)$, $\Delta^-\hat{H}(\hat{w}) = \hat{H}(w_- - z) - \hat{H}(w - z)$ and

$$\begin{aligned}\hat{w}_+^*(\hat{w}) &= \arg\max_{\hat{w}_+ \geq \hat{w}} \left(\hat{J}(\hat{w}_+) - \hat{J}(\hat{w})\right)^{1-\chi}\left(\hat{H}(\hat{w}_+) - \hat{H}(\hat{w})\right)^{\chi} \\ \hat{w}_-^*(\hat{w}) &= \arg\max_{\hat{w}_- \leq \hat{w}} \left(\hat{J}(\hat{w}_-) - \hat{J}(\hat{w})\right)^{1-\chi}\left(\hat{H}(\hat{w}_-) - \hat{H}(\hat{w})\right)^{\chi}.\end{aligned}$$

6. Optimal continuation regions:

$$\begin{aligned}\hat{\mathcal{W}}^{h*} &= \{\hat{w} : \hat{H}(\hat{w}) > 0 \text{ or } e^{\hat{w}} > \hat{\rho}\hat{U}\}, \\ \hat{\mathcal{W}}^{j*} &= \{\hat{w} : \hat{J}(\hat{w}) > 0 \text{ or } 1 - e^{\hat{w}} > 0\}.\end{aligned}$$

Solution Algorithm:

1. Define equidistant grids $\hat{\mathcal{H}} = \{\hat{w}_1, \dots, \hat{w}_{N_w}\}$, where the $\Delta_w \equiv \hat{w}_i - \hat{w}_{i-1}$.
2. Guess $\hat{U}^0$, $\hat{J}^0(\hat{w})$ and $\hat{H}^0(\hat{w})$. Proposal: as a starting point, choose the solution of the baseline model without on-the-job search and fixed wages, which has a closed-form solution.
3. Suppose we are in iteration $\mathtt{n}$ with guess $\hat{U}^{\mathtt{n}}$, $\hat{J}^{\mathtt{n}}(\hat{w})$ and $\hat{H}^{\mathtt{n}}(\hat{w})$:

   3.1 Compute the continuation regions $\hat{\mathcal{W}}^{h\mathtt{n}} = \{\hat{w} : \hat{H}^{\mathtt{n}}(\hat{w}) > 0 \; or \; e^{\hat{w}} > \hat{\rho}\hat{U}^{\mathtt{n}}\}$ and $\hat{\mathcal{W}}^{j\mathtt{n}} = \{\hat{w} : \hat{J}^{\mathtt{n}}(\hat{w}) > 0 \; or \; 1 - e^{\hat{w}} > 0\}$.

   3.2 From the free-entry condition solve for $\hat{\theta}^{\mathtt{n}}(\hat{w}_i)$ and $f(\hat{\theta}^{\mathtt{n}}(\hat{w}_i))$ for each $\hat{w}_i$ in the grid.

   3.3 Solve the search problem by computing $s^{*\mathtt{n}}(\hat{w}_i) = \left(\frac{f(\hat{\theta}^{\mathtt{n}}(\hat{w}^{*\mathtt{n}}_{jj}(\hat{w}_i)))[\hat{H}^{\mathtt{n}}(\hat{w}^{*\mathtt{n}}_{jj}(\hat{w}_i)) - \hat{H}^{\mathtt{n}}(\hat{w}_i)]}{\mu}\right)^{\phi}$ and $\hat{w}^{*\mathtt{n}}_{jj}(\hat{w}_i) = \arg\max_{w_{jj}}$ $f(\hat{\theta}^{\mathtt{n}}(\hat{w}_{jj}))[\hat{H}^{\mathtt{n}}(\hat{w}_{jj}) - \hat{H}^{\mathtt{n}}(\hat{w}_i)]$ for each $\hat{w}_i$ in the grid. Define the $N_w \times N_w$ transition matrix based on optimal search strategy when employed as

$$\boldsymbol{\mathcal{A}}^{\mathtt{n}}_{jj\,l,m} = \begin{cases} 1, & \text{if } \hat{w}_i = \hat{w}_l \text{ and } \hat{w}^{*}_{jj}(\hat{w}_i) = \hat{w}_m \\ 0, & \text{otherwise.} \end{cases}$$

   3.4 Solve the bargaining problem by computing

$$\hat{w}^{*\mathtt{n}}_b(\hat{w}_i) = \arg\max_{\hat{w}_b} \left(\hat{J}^{\mathtt{n}}(\hat{w}_b) - \hat{J}^{\mathtt{n}}(\hat{w}_i)\right)^{1-\chi} \left(\hat{H}^{\mathtt{n}}(\hat{w}_b) - \hat{H}^{\mathtt{n}}(\hat{w}_i)\right)^{\chi}$$

   for each $\hat{w}_i$ in the grid. Define the $N_w \times N_w$ transition matrix based on the bargaining solution as

$$\boldsymbol{\mathcal{A}}^{\mathtt{n}}_{b\,l,m} = \begin{cases} 1, & \text{if } \hat{w}_i = \hat{w}_l \text{ and } \hat{w}^{*\mathtt{n}}_{b}(\hat{w}_i) = \hat{w}_m \\ 0, & \text{otherwise.} \end{cases}$$

   3.5 Update worker's value by solving a linear complementarity problem (LCP). The discretized version of their HJBVI equation is given by

$$\begin{aligned} \hat{\rho}\hat{H}(\hat{w}_i) = \max\Big\{0\,,\, & e^{\hat{w}_i} - \hat{\rho}\hat{U} - \hat{\gamma}\frac{\partial \hat{H}(\hat{w}_i)}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 \hat{H}(\hat{w}_i)}{\partial \hat{w}^2} - \delta\hat{H}(\hat{w}_i) \\ & + \beta^+\Delta^+\hat{H}(\hat{w}_i)G^+(\Delta^+\hat{J}(\hat{w}_i)) + \beta^-\Delta^-\hat{H}(\hat{w}_i)G^-(\Delta^-\hat{J}(\hat{w}_i)) \\ & + s^*(\hat{w}_i)f(\hat{\theta}(\hat{w}^*_{jj}(\hat{w}_i)))[\hat{H}(\hat{w}^*_{jj}(\hat{w}_i)) - \hat{H}(\hat{w}_i)] - \frac{\mu\,(s^*(\hat{w}_i))^{1+1/\phi}}{1+1/\phi}\Big\},\ \forall \hat{w}_i \in \hat{\mathcal{W}}^{j\mathtt{n}} \end{aligned}$$

   and $\hat{H}(\hat{w}_i) = 0 \quad \forall \hat{w}_i \notin \hat{\mathcal{W}}^{j\mathtt{n}}$.

   The derivatives are discretized in the following way: for a given function $\mathcal{F}$, define forward and backward first differences and second differences as

$$\begin{aligned} \mathcal{F}^F_{\hat{w}}(\hat{w}_i) &\approx \frac{\mathcal{F}(\hat{w}_{i+1}) - \mathcal{F}(\hat{w}_i)}{\Delta_w} \\ \mathcal{F}^B_{\hat{w}}(\hat{w}_i) &\approx \frac{\mathcal{F}(\hat{w}_i) - \mathcal{F}(\hat{w}_{i-1})}{\Delta_w} \\ \mathcal{F}_{\hat{w}\hat{w}}(\hat{w}_i) &\approx \frac{\mathcal{F}(\hat{w}_{i+1}) - 2\mathcal{F}(\hat{w}_i) + \mathcal{F}(\hat{w}_{i-1})}{(\Delta_w)^2}. \end{aligned}$$

The HJBVI of the worker in matrix notation is given by

$$\min \Bigg\{ \frac{\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \hat{\boldsymbol{H}}^{\mathrm{n}}}{\Delta t} + \hat{\rho}\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\}\Big(e^{\hat{\boldsymbol{w}}} - \hat{\rho}\hat{\boldsymbol{U}} - \frac{\mu\,(\boldsymbol{s}^{*\mathrm{n}})^{1+1/\phi}}{1+1/\phi} + \boldsymbol{\mathcal{A}}_{\boldsymbol{z}}\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \delta\hat{\boldsymbol{H}}^{\mathrm{n}+1}$$
$$+ \,diag\left(\beta^{+}G^{+}(\Delta^{+}\hat{\boldsymbol{J}}^{\mathrm{n}})\mathbf{1}\{\hat{\boldsymbol{w}}_{\boldsymbol{b}}^{*\mathrm{n}} > \hat{\boldsymbol{w}}\} + \beta^{-}G^{-}(\Delta^{-}\hat{\boldsymbol{J}}^{\mathrm{n}})\mathbf{1}\{\hat{\boldsymbol{w}}_{\boldsymbol{b}}^{*\mathrm{n}}(\hat{\boldsymbol{w}}) < \hat{\boldsymbol{w}}\}\right)\left[\boldsymbol{\mathcal{A}}_{\boldsymbol{b}}^{\mathrm{n}}\hat{\boldsymbol{H}}^{\mathrm{n}} - \hat{\boldsymbol{H}}^{\mathrm{n}}\right]$$
$$+ \,diag(\boldsymbol{s}^{*\mathrm{n}})\left[\boldsymbol{\mathcal{A}}_{\boldsymbol{jj}}^{\mathrm{n}}\left(diag(f(\boldsymbol{\theta}^{\mathrm{n}}))\hat{\boldsymbol{H}}^{\mathrm{n}+1}\right) - diag\left(\boldsymbol{\mathcal{A}}_{jj}f(\boldsymbol{\theta}^{\mathrm{n}})\right)\hat{\boldsymbol{H}}^{\mathrm{n}+1}\right]\Big), \hat{\boldsymbol{H}}^{\mathrm{n}+1}\Bigg\} = 0 \tag{A.1}$$

Here, $diag(x)$ is a square matrix with vector x in its diagonal. Also, $\boldsymbol{\mathcal{A}}_{\boldsymbol{z}}$ is the discretized transition probability matrix for the process $d\hat{w}$. To construct this matrix we follow an upwind scheme. Thus, row i is given by

$$\boldsymbol{\mathcal{A}}_{\boldsymbol{z}[i,]}\hat{\boldsymbol{H}} = \left(\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)\hat{H}(\hat{w}_{i-1}) + \left(-\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\hat{\gamma}^{-}}{\Delta_w} - \frac{\sigma^2}{(\Delta_w)^2}\right)\hat{H}(\hat{w}_i) + \left(-\frac{\hat{\gamma}^{-}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)\hat{H}(\hat{w}_{i+1})$$

where $x^{+} \equiv \max\{x, 0\}$ and $x^{-} \equiv \min\{x, 0\}$. At the lower boundary, we have the reflecting barrier $\hat{H}^{B}_{\hat{w}}(\hat{w}_1) \approx \frac{\hat{H}(\hat{w}_1) - \hat{H}(\hat{w}_0)}{\Delta_w} = 0$ and hence $\hat{H}(\hat{w}_0) = \hat{H}(\hat{w}_1)$. Thus,

$$\boldsymbol{\mathcal{A}}_{\boldsymbol{z}[1,]}\hat{\boldsymbol{H}} = \left(\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)\hat{H}(\hat{w}_1) + \left(-\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\hat{\gamma}^{-}}{\Delta_w} - \frac{\sigma^2}{(\Delta_w)^2}\right)\hat{H}(\hat{w}_1) + \left(-\frac{\hat{\gamma}^{-}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)\hat{H}(\hat{w}_2)$$

Similarly, at the upper boundary, we have the reflecting barrier $\hat{H}^{F}_{\hat{w}}(\hat{w}_{N_w}) \approx \frac{\hat{H}(\hat{w}_{N_w+1}) - \hat{H}(\hat{w}_{N_w})}{\Delta_w} = 0$ and hence $\hat{H}(\hat{w}_{N_w}) = \hat{H}(\hat{w}_{N_w+1})$. Thus,

$$\boldsymbol{\mathcal{A}}_{\boldsymbol{z}[N_w,]}\hat{\boldsymbol{H}} = \left(\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)\hat{H}(\hat{w}_{N_w-1}) + \left(-\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\hat{\gamma}^{-}}{\Delta_w} - \frac{\sigma^2}{(\Delta_w)^2}\right)\hat{H}(\hat{w}_{N_w}) + \left(-\frac{\hat{\gamma}^{-}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)\hat{H}(\hat{w}_{N_w}).$$

To summarize, let $a_L \equiv \left(\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)$, $a_M \equiv \left(-\frac{\hat{\gamma}^{+}}{\Delta_w} + \frac{\hat{\gamma}^{-}}{\Delta_w} - \frac{\sigma^2}{(\Delta_w)^2}\right)$ and $a_H \equiv \left(-\frac{\hat{\gamma}^{-}}{\Delta_w} + \frac{\sigma^2}{2(\Delta_w)^2}\right)$. Then, we can write the $N_w \times N_w$ matrix as

$$\boldsymbol{\mathcal{A}}_{z} = \begin{bmatrix} a_L + a_M & a_H & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ a_L & a_M & a_H & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & a_L & a_M & a_H & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & a_L & a_M & a_H \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & a_L & a_M + a_H \end{bmatrix}$$

Next, (3e) can be rewritten as:

$$\left(\hat{\boldsymbol{H}}^{\mathrm{n}+1}\right)^{T}\left(\frac{\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \hat{\boldsymbol{H}}^{\mathrm{n}}\mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\}}{\Delta t} + \hat{\rho}\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\}\Big(e^{\hat{\boldsymbol{w}}} - \hat{\rho}\hat{\boldsymbol{U}} - \frac{\mu\,(\boldsymbol{s}^{*\mathrm{n}})^{1+1/\phi}}{1+1/\phi} + \boldsymbol{\mathcal{A}}_{\boldsymbol{W}}^{\mathrm{n}}\hat{\boldsymbol{H}}^{\mathrm{n}+1}\Big)\right) = 0$$

$$\hat{\boldsymbol{H}}^{\mathrm{n}+1} \geq 0$$

$$\frac{\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \hat{\boldsymbol{H}}^{\mathrm{n}}\mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\}}{\Delta t} + \hat{\rho}\hat{\boldsymbol{H}}^{\mathrm{n}+1} - \mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\}\Big(e^{\hat{\boldsymbol{w}}} - \hat{\rho}\hat{\boldsymbol{U}} - \frac{\mu\,(\boldsymbol{s}^{*\mathrm{n}})^{1+1/\phi}}{1+1/\phi} + \boldsymbol{\mathcal{A}}_{\boldsymbol{W}}^{\mathrm{n}}\hat{\boldsymbol{H}}^{\mathrm{n}+1}\Big) \geq 0,$$

where

$$\boldsymbol{\mathcal{A}}_W^{\mathrm{n}} := -diag(\delta) + \boldsymbol{\mathcal{A}}_z + diag(\boldsymbol{s}^{*\mathrm{n}}) \left[\boldsymbol{\mathcal{A}}_{jj}^{\mathrm{n}} diag(f(\boldsymbol{\theta}^{\mathrm{n}})) - diag\left(\boldsymbol{\mathcal{A}}_{jj}^{\mathrm{n}} f(\boldsymbol{\theta}^{\mathrm{n}})\right)\right]$$
$$+ diag\left(\beta^+ G^+(\Delta^+ \hat{\boldsymbol{J}}^{\mathrm{n}}) \mathbf{1}\{\hat{\boldsymbol{w}}_b^{*\mathrm{n}} > \hat{\boldsymbol{w}}\} + \beta^- G^-(\Delta^- \hat{\boldsymbol{J}}^{\mathrm{n}}) \mathbf{1}\{\hat{\boldsymbol{w}}_b^{*\mathrm{n}} < \hat{\boldsymbol{w}}\}\right) \left[\boldsymbol{\mathcal{A}}_b^{\mathrm{n}} - \boldsymbol{I}\right].$$

This is a LCP with $\zeta = \hat{\boldsymbol{H}}^{\mathrm{n}+1}$, $M = \left(\hat{\rho} + \frac{1}{\Delta t}\right) \boldsymbol{I} - \mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\} \boldsymbol{\mathcal{A}}_W^{\mathrm{n}}$, $q = -\mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{j\mathrm{n}}\}\left(e^{\hat{\boldsymbol{w}}} - \hat{\rho}\hat{\boldsymbol{U}} - \frac{\mu(\boldsymbol{s}^{*\mathrm{n}})^{1+1/\phi}}{1+1/\phi} + \frac{\hat{\boldsymbol{H}}^{\mathrm{n}}}{\Delta t}\right)$ satisfying

$$\zeta^T(M\zeta + q) = 0$$
$$\zeta \geq 0$$
$$M\zeta + q \geq 0$$

Use an LCP solver to find $\hat{H}^{\mathrm{n}+1}(\hat{w})$ in the entire domain.

3.6 Update worker's continuation region: $\hat{\mathcal{W}}^{h\mathrm{n}+1} = \{\hat{w} : \hat{H}^{\mathrm{n}+1}(\hat{w}) > 0 \text{ } or \text{ } e^{\hat{w}} > \hat{\rho}\hat{U}^{\mathrm{n}}\}$.

3.7 Solve the LCP of the firm. The HJBVI of the firm in matrix notation is given by

$$\min\left\{\frac{\hat{\boldsymbol{J}}^{\mathrm{n}+1} - \hat{\boldsymbol{J}}^{\mathrm{n}}\mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{h\mathrm{n}}\}}{\Delta t} + \hat{\rho}\hat{\boldsymbol{J}}^{\mathrm{n}+1} - \mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{h\mathrm{n}}\}\left(1 - e^{\hat{\boldsymbol{w}}} + \boldsymbol{\mathcal{A}}_z\hat{\boldsymbol{J}}^{\mathrm{n}+1} - \delta\hat{\boldsymbol{J}}^{\mathrm{n}+1} - diag(\boldsymbol{s}^{*\mathrm{n}}) diag\left(\boldsymbol{\mathcal{A}}_{jj}^{\mathrm{n}} f(\boldsymbol{\theta}^{\mathrm{n}})\right)\hat{\boldsymbol{J}}^{\mathrm{n}+1}\right)\right.$$
$$\left. + diag\left(\beta^+ G^+(\Delta^+ \hat{\boldsymbol{J}}^{\mathrm{n}}) \mathbf{1}\{\hat{\boldsymbol{w}}_b^{*\mathrm{n}} > \hat{\boldsymbol{w}}\} + \beta^- G^-(\Delta^- \hat{\boldsymbol{J}}^{\mathrm{n}}) \mathbf{1}\{\hat{\boldsymbol{w}}_b^{*\mathrm{n}} < \hat{\boldsymbol{w}}\}\right)\left[\boldsymbol{\mathcal{A}}_b\hat{\boldsymbol{J}}^{\mathrm{n}} - \hat{\boldsymbol{J}}^{\mathrm{n}}\right], \hat{\boldsymbol{J}}^{\mathrm{n}+1}\right\} = 0.$$

Solve the LCP problem with $\zeta = \hat{\boldsymbol{J}}^{\mathrm{n}+1}$, $M = \left(\hat{\rho} + \frac{1}{\Delta t}\right)\boldsymbol{I} - \mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^{h\mathrm{n}}\}\boldsymbol{\mathcal{A}}_J^{\mathrm{n}}$ with $\boldsymbol{\mathcal{A}}_J^{\mathrm{n}} = -diag(\delta) + \boldsymbol{\mathcal{A}}_z - diag(\boldsymbol{s}^{*\mathrm{n}}) diag\left(\boldsymbol{\mathcal{A}}_{jj}^{\mathrm{n}} f(\boldsymbol{\theta}^{\mathrm{n}})\right) + diag\left(\beta^+ G^+(\Delta^+ \hat{\boldsymbol{J}}^{\mathrm{n}})\mathbf{1}\{\hat{\boldsymbol{w}}_b^* > \hat{\boldsymbol{w}}\} + \beta^- G^-(\Delta^- \hat{\boldsymbol{J}}^{\mathrm{n}})\mathbf{1}\{\hat{\boldsymbol{w}}_b^* < \hat{\boldsymbol{w}}\}\right)\left[\boldsymbol{\mathcal{A}}_b - \boldsymbol{I}\right]$, $q = -\mathbf{1}\{\hat{\boldsymbol{w}} \in \hat{\mathcal{W}}^h\}\left(1 - e^{\hat{\boldsymbol{w}}} + \frac{\hat{\boldsymbol{J}}^{\mathrm{n}}}{\Delta t}\right)$. Use a LCP solver to find $\hat{J}^{\mathrm{n}+1}(\hat{w})$ in the entire domain.

3.8 Update firm's continuation region: $\hat{\mathcal{W}}^{j\mathrm{n}+1} = \{\hat{w} : \hat{J}^{\mathrm{n}+1}(\hat{w}) > 0 \text{ } or \text{ } 1 - e^{\hat{w}} > 0\}$.

3.9 Compute the optimal search strategy when unemployed by finding

$$\hat{w}_u^{*\mathrm{n}+1} = \arg\max_{\hat{w}_u} \theta^{\mathrm{n}}(\hat{w}_u)^{1-\alpha}\hat{H}^{\mathrm{n}}(\hat{w}_u)$$

3.10 Compute the new unemployment value from

$$\frac{\hat{U}^{\mathrm{n}+1} - \hat{U}^{\mathrm{n}}}{\Delta t} + \hat{\rho}\hat{U}^{\mathrm{n}+1} = \tilde{B} + \theta^{\mathrm{n}}(\hat{w}_u^{*\mathrm{n}+1})^{1-\alpha}\hat{H}^{\mathrm{n}}(\hat{w}_u^{*\mathrm{n}+1}).$$

3.11 If $||\hat{U}^{\mathrm{n}+1} - \hat{U}^{\mathrm{n}}|| < tol_U$, $||\hat{H}^{\mathrm{n}+1} - \hat{H}^{\mathrm{n}}|| < tol_W$ and $||\hat{J}^{\mathrm{n}+1} - \hat{J}^{\mathrm{n}}|| < tol_J$, stop. Otherwise, go back to Step 3 with guess $\hat{U}^{\mathrm{n}+1}$, $\hat{H}^{\mathrm{n}+1}$ and $\hat{J}^{\mathrm{n}+1}$.

A.2 Solving the Kolmogorov Forward Equation

Problem: Find $g^h(\hat{w})$ and $g^u(\hat{w})$ such that

$$\left(\delta + s(\hat{w})f(\hat{w}_{jj}^*(\hat{w})) + \tilde{\beta}(\hat{w})\right)g^h(\hat{w}) = -(-\gamma)(g^h)'(\hat{w}) + \frac{\sigma^2}{2}(g^h)''(\hat{w})$$

$$
+\int \left[1(\hat{w}^*_{jj}(x)=\hat{w})s(x)f(\hat{w})+1(\hat{w}^*_b(x)=\hat{w})\tilde{\beta}(x)\right] g^h(x)\,\mathrm{d}x \quad \forall \hat{w}\in(\hat{\mathcal{W}}^{j*}\cap\hat{\mathcal{W}}^{h*})\backslash\{\hat{w}^*_u\},
$$

$$
f(\hat{w}^*_u)g^u(\hat{w}) = -(-\gamma)(g^u)'(\hat{w})+\frac{\sigma^2}{2}(g^u)''(\hat{w}) \quad \forall \hat{w}\in\mathbb{R}\backslash\{\hat{w}^*_u\},
$$

and

$$
\begin{aligned}
g^h(\hat{w}^-) &= g^h(\hat{w}^+) = 0,\\
\lim_{\hat{w}\to-\infty} g^u(\hat{w}) &= \lim_{\hat{w}\to\infty} g^u(\hat{w}) = 0,\\
1 &= \int_{-\infty}^{\infty} g^u(\hat{w})\,\mathrm{d}\hat{w} + \int_{\hat{w}^-}^{\hat{w}^+} g^h(\hat{w})\,\mathrm{d}\hat{w},\\
f(\hat{w}^*_u)(1-\mathcal{E}) &= \delta\mathcal{E}+\frac{\sigma^2}{2}\left[\lim_{\hat{w}\downarrow\hat{w}^-}(g^h)'(\hat{w})-\lim_{\hat{w}\uparrow\hat{w}^+}(g^h)'(\hat{w})\right],
\end{aligned}
$$

where $\tilde{\beta}(x) := \beta^+G^+(\Delta^+\hat{J}(x))+\beta^-G^-(\Delta^-\hat{J}(x))$. To solve this, we can discretize each equation. Let

$$
\begin{aligned}
\boldsymbol{\mathcal{A}}_W := &-diag(\delta)+\boldsymbol{\mathcal{A}}_z+diag(\boldsymbol{s}^*)\left[\boldsymbol{\mathcal{A}}_{jj}diag(f(\boldsymbol{\theta}))-diag\left(\boldsymbol{\mathcal{A}}_{jj}f(\boldsymbol{\theta})\right)\right]\\
&+diag\left(\beta^+G^+(\Delta^+\hat{\boldsymbol{J}}^*)\mathbf{1}\{\hat{\boldsymbol{w}}^*_b>\hat{\boldsymbol{w}}\}+\beta^-G^-(\Delta^-\hat{\boldsymbol{J}}^*)\mathbf{1}\{\hat{\boldsymbol{w}}^*_b<\hat{\boldsymbol{w}}\}\right)\left[\boldsymbol{\mathcal{A}}_b-\boldsymbol{I}\right]
\end{aligned}
$$

Here, the operator $\boldsymbol{\mathcal{A}}_z$ should have a reflecting barrier. Note that, relative to the solution of the HJBVI equations, the $\boldsymbol{\mathcal{A}}_z$ operator uses γ and not $\hat{\gamma}$. Let $\boldsymbol{I}_x$ be the diagonal matrix with entries equal to one if condition x is satisfied. Then, the above system of equations can be expressed as

$$
\begin{bmatrix}
\boldsymbol{I}_{\hat{w}_j\in(\hat{\mathcal{W}}^{j*}\cap\hat{\mathcal{W}}^{h*})\backslash\{\hat{w}^*_u\}}\mathcal{A}_W^T+\boldsymbol{I}_{\hat{w}_j\notin(\hat{\mathcal{W}}^{j*}\cap\hat{\mathcal{W}}^{h*})\backslash\{\hat{w}^*_u\}} & \mathbf{0}\\
\mathbf{0} & \boldsymbol{I}_{\hat{w}_j\neq\hat{w}^*_u}\left(\boldsymbol{\mathcal{A}}_z^T-f(\hat{w}^*_u)\boldsymbol{I}\right)\\
[\Delta w,\dots,\Delta w] & [\Delta w,\dots,\Delta w]\\
-\left(\delta\Delta_w[1,\dots,1]+\frac{\sigma^2}{2\Delta_w}\left[[0,\dots,0,-1\{\hat{w}_j=\hat{w}^-\},1,0,\dots,0]-[0,\dots,0,-1,1\{\hat{w}_j=\hat{w}^+\},0,\dots,0]\right]\right) & f(\hat{w}^*_u)\Delta_w[1,\dots,1]
\end{bmatrix}
\begin{bmatrix}\boldsymbol{g}^h\\ \boldsymbol{g}^u\end{bmatrix}
=
\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ 1\\ 0\end{bmatrix}
$$

Use a linear solver to solve for $\boldsymbol{g}^h$ and $\boldsymbol{g}^u$.[7]

A.3 Solving for Transition Dynamics

Summary of equations: Define the transformed drift $\hat{\gamma} := \gamma+\sigma^2$ and the transformed discount factor $\hat{\rho} := \rho-\gamma-\sigma^2/2$.

1. Free entry condition for $\hat{\theta}(\hat{w},t)$:

$$
\tilde{K} = \hat{\theta}(\hat{w},t)^{-\alpha}\hat{J}(\hat{w},t).
$$

2. Normalized value of unemployment:

$$
\hat{\rho}\hat{U}(t) = \tilde{B}+\hat{\theta}(\hat{w}^*_u(t))^{1-\alpha}\hat{H}(\hat{w}^*_u(t),t)+\frac{\partial\hat{U}(t)}{\partial t},
$$

   where $\hat{w}^*_u(t) = \arg\max_{\hat{w}_u}\hat{\theta}(\hat{w}_u,t)^{1-\alpha}\hat{H}(\hat{w}_u,t)$.

3. Normalized value function of an employed worker:

[7] Note that the boundary conditions for $\boldsymbol{g}^u$ are not included in the above equations, because they are replaced by the reflecting barrier in $\boldsymbol{\mathcal{A}}_z$.

$$\hat{\rho}\hat{H}(\hat{w},t) = \max\Big\{0\,,\, e^{\hat{w}} - \hat{\rho}\hat{U}(t) - \frac{\mu\left(s^*(\hat{w},t)\right)^{1+1/\phi}}{1+1/\phi} - \delta\hat{H}(\hat{w},t) - \hat{\gamma}\frac{\partial\hat{H}(\hat{w},t)}{\partial\hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2\hat{H}(\hat{w},t)}{\partial\hat{w}^2}$$
$$+ \left(\beta^+G^+(\Delta^+\hat{J}(\hat{w},t))\mathbf{1}\{\hat{w}_b^*(\hat{w},t) > \hat{w}\} + \beta^-G^-(\Delta^-\hat{J}(\hat{w},t))\mathbf{1}\{\hat{w}_b^*(\hat{w},t) < \hat{w}\}\right)\left[\hat{H}(\hat{w}_b^*,t) - \hat{H}(\hat{w},t)\right]$$
$$+ s^*(\hat{w},t)f(\hat{\theta}(\hat{w}_{jj}^*(\hat{w},t),t))[\hat{H}(\hat{w}_{jj}^*(\hat{w},t),t) - \hat{H}(\hat{w},t)] + \frac{\partial\hat{H}(\hat{w},t)}{\partial t}\Big\},\ \forall\hat{w}\in\hat{\mathcal{W}}_t^{j*}$$

and $\hat{H}(\hat{w},t) = 0\ \ \forall\hat{w}\notin\hat{\mathcal{W}}_t^{j*}$, where $s^*(\hat{w},t) = \left(\frac{f(\hat{\theta}(\hat{w}_{jj}^*(\hat{w},t)))[\hat{H}(\hat{w}_{jj}^*(\hat{w},t),t)-\hat{H}(\hat{w},t)]}{\mu}\right)^{\phi}$ and $\hat{w}_{jj}^*(\hat{w},t) = \arg\max_{w_{jj}} f(\theta(\hat{w}_{jj},t))[\hat{H}(\hat{w}_{jj},t) - \hat{H}(\hat{w},t)]$ and $\hat{w}_b^*(\hat{w},t) = \arg\max_{\hat{w}_b}\left(\hat{J}(\hat{w}_b,t) - \hat{J}(\hat{w},t)\right)^{\chi}\left(\hat{H}(\hat{w}_b,t) - \hat{H}(\hat{w},t)\right)^{1-\chi}$.

4. Normalized value function of a filled vacancy:

$$\hat{\rho}\hat{J}(\hat{w},t) = \max\Big\{0\,,\,1 - e^{\hat{w}} - \delta\hat{J}(\hat{w},t) - \hat{\gamma}\frac{\partial\hat{J}(\hat{w},t)}{\partial\hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2\hat{J}(\hat{w},t)}{\partial\hat{w}^2}$$
$$+ \left(\beta^+G^+(\Delta^+\hat{J}(\hat{w},t))\mathbf{1}\{\hat{w}_b^*(\hat{w},t) > \hat{w}\} + \beta^-G^-(\Delta^-\hat{J}(\hat{w},t))\mathbf{1}\{\hat{w}_b^*(\hat{w},t) < \hat{w}\}\right)\left[\hat{J}(\hat{w}_b^*,t) - \hat{J}(\hat{w},t)\right]$$
$$- s^*(\hat{w},t)f(\hat{\theta}(\hat{w}_{jj}^*(\hat{w},t)))\hat{J}(\hat{w},t) + \frac{\partial\hat{J}(\hat{w},t)}{\partial t}\Big\},\ \forall\hat{w}\in\hat{\mathcal{W}}_t^{h*}$$

and $\hat{J}(\hat{w},t) = 0\ \ \forall\hat{w}\notin\hat{\mathcal{W}}_t^{h*}$.

5. Optimal bargaining:

$$\hat{w}_+^*(\hat{w}) = \arg\max_{\hat{w}_+\geq\hat{w}}\left(\hat{J}(\hat{w}_+) - \hat{J}(\hat{w})\right)^{1-\chi}\left(\hat{H}(\hat{w}_+) - \hat{H}(\hat{w})\right)^{\chi}$$
$$\hat{w}_-^*(\hat{w}) = \arg\max_{\hat{w}_-\leq\hat{w}}\left(\hat{J}(\hat{w}_-) - \hat{J}(\hat{w})\right)^{1-\chi}\left(\hat{H}(\hat{w}_-) - \hat{H}(\hat{w})\right)^{\chi}.$$

6. Optimal continuation regions:

$$\hat{\mathcal{W}}_t^{h*} = \{\hat{w} : \hat{H}(\hat{w},t) > 0 \text{ or } e^{\hat{w}} > \hat{\rho}\hat{U}(t)\},$$
$$\hat{\mathcal{W}}_t^{j*} = \{\hat{w} : \hat{J}(\hat{w},t) > 0 \text{ or } 1 - e^{\hat{w}} > 0\}.$$

Solution Algorithm:

1. Define equidistant grids $\hat{\mathcal{H}} = \{\hat{w}_1,\ldots,\hat{w}_{N_w}\}$, where the $\Delta_w \equiv \hat{w}_i - \hat{w}_{i-1}$. Then, define the time step $\Delta t \equiv (\Delta w/\sigma)^2$.
2. Fix a terminal time T and impose $\hat{H}(\hat{w},T) = \hat{H}(\hat{w})$, $\hat{J}(\hat{w},T) = \hat{J}(\hat{w})$ and $\hat{U}(T) = \hat{U}$.
3. Solve the value functions by backward induction. Suppose we are finding the solution for period $0 \leq t < T$:

   3.1 Solve the game between the employed worker and the firm using the implicit method (and using the notation $t+1 \equiv t + \Delta t$). The value of the worker can be written as

$$\hat{\rho}\boldsymbol{\hat{H}_t} = \max\Big\{0\,,\,e^{\boldsymbol{\hat{w}}} - \hat{\rho}\hat{U}_{t+1} - \frac{\mu\left(s_{t+1}^*(\boldsymbol{\hat{w}})\right)^{1+1/\phi}}{1+1/\phi} + \frac{\boldsymbol{\hat{H}_{t+1}} - \boldsymbol{\hat{H}_t}}{\Delta t} + \boldsymbol{\mathcal{A}_{W,t+1}}\boldsymbol{\hat{H}_t}\Big\},\ \forall\hat{w}_i\in\hat{\mathcal{W}}_t^{j*}$$
$$\hat{\rho}\boldsymbol{\hat{H}_t} = 0\ \forall\hat{w}_i\notin\hat{\mathcal{W}}_t^{j*},$$

   where

$$\boldsymbol{\mathcal{A}_{W,t+1}} \equiv -diag(\delta) + \boldsymbol{\mathcal{A}_z} + diag(\boldsymbol{s}_{t+1}^*)\left[\boldsymbol{\mathcal{A}_{jj,t+1}}diag(f(\boldsymbol{\theta_{t+1}})) - diag\left(\boldsymbol{\mathcal{A}_{jj,t+1}}f(\boldsymbol{\theta_{t+1}})\right)\right]$$

$$+\left(\beta^{+}G^{+}(\Delta^{+}\hat{\boldsymbol{J}}_{\boldsymbol{t}})\mathbf{1}\{\hat{w}^{*}_{b,t+1}>\hat{w}\}+\beta^{-}G^{-}(\Delta^{-}\hat{\boldsymbol{J}}_{\boldsymbol{t}})\mathbf{1}\{\hat{w}^{*}_{b,t+1}<\hat{w}\}\right)\left[\boldsymbol{\mathcal{A}}_{\boldsymbol{b,t+1}}-\boldsymbol{I}\right],$$

with

$$\hat{w}^{*}_{jj,t+1}(\hat{w}_i)=\arg\max_{\hat{w}_{jj}} f\left(\theta_{t+1}(\hat{w}_{jj})\right)\left[\hat{H}_{t+1}(\hat{w}_{jj})-\hat{H}_{t+1}(\hat{w}_i)\right]$$

$$s^{*}_{t+1}(\hat{w}_i)=\left(\frac{f\left(\theta_{t+1}(\hat{w}^{*}_{jj,t+1}(\hat{w}_i))\right)\left[\hat{H}_{t+1}(\hat{w}^{*}_{jj,t+1}(\hat{w}_i))-\hat{H}_{t+1}(\hat{w}_i)\right]}{\mu}\right)^{\phi}$$

$$\boldsymbol{\mathcal{A}}_{\boldsymbol{jj,t+1}_{l,m}}=\begin{cases}1, & \text{if } \hat{w}_i=\hat{w}_l \text{ and } \hat{w}^{*}_{jj,t+1}(\hat{w}_i)=\hat{w}_m\\ 0, & \text{otherwise.}\end{cases}$$

$$\hat{w}^{*}_{b,t+1}(\hat{w}_i)=\arg\max_{\hat{w}_b}\left[\hat{J}_{t+1}(\hat{w}_b)-\hat{J}_{t+1}(\hat{w}_i)\right]^{1-\chi}\left[\hat{H}_{t+1}(\hat{w}_b)-\hat{H}_{t+1}(\hat{w}_i)\right]^{\chi}$$

$$\boldsymbol{\mathcal{A}}_{\boldsymbol{b,t+1}_{l,m}}=\begin{cases}1, & \text{if } \hat{w}_i=\hat{w}_l \text{ and } \hat{w}^{*}_{b,t+1}(\hat{w}_i)=\hat{w}_m\\ 0, & \text{otherwise.}\end{cases}$$

$$\theta_{t+1}(\hat{w}_i)=\left(\frac{\hat{J}_{t+1}(\hat{w}_i)}{\tilde{K}}\right)^{1/\alpha}.$$

This is a LCP with $\zeta\equiv\hat{\boldsymbol{H}}_{\boldsymbol{t}}$, $M\equiv\left(\hat{\rho}+\frac{1}{\Delta t}\right)\boldsymbol{I}-\mathbf{1}\{\hat{\boldsymbol{w}}\in\hat{\mathcal{Z}}^{j}_{t+1}\}\boldsymbol{\mathcal{A}}_{\mathbf{W,t+1}}$, $q\equiv-\mathbf{1}\{\hat{\boldsymbol{w}}\in\hat{\mathcal{Z}}^{j}_{t+1}\}\left(e^{\hat{\boldsymbol{w}}}-\hat{\rho}\hat{\boldsymbol{U}}_{\boldsymbol{t+1}}-\frac{\mu\left(s^{*}_{t+1}\right)^{1+1/\phi}}{1+1/\phi}+\frac{\hat{\boldsymbol{H}}_{\boldsymbol{t+1}}}{\Delta t}\right)$. Similarly, the value of the firm can be written as

$$\hat{\rho}\hat{\boldsymbol{J}}_{\boldsymbol{t}}=\max\left\{0\,,\,1-e^{\hat{\boldsymbol{w}}}+\frac{\hat{\boldsymbol{J}}_{\boldsymbol{t+1}}-\hat{\boldsymbol{J}}_{\boldsymbol{t}}}{\Delta t}+\boldsymbol{\mathcal{A}}_{\boldsymbol{J,t+1}}\hat{\boldsymbol{J}}_{\boldsymbol{t}}\right\},\ \forall\hat{w}_i\in\hat{\mathcal{C}}^{h}_{t}$$

$$\hat{\rho}\hat{\boldsymbol{J}}_{\boldsymbol{t}}=0\ \forall\hat{w}_i\in(\hat{\mathcal{C}}^{h}_{t})^{c},$$

where

$$\boldsymbol{\mathcal{A}}_{\boldsymbol{J,t+1}}\equiv-diag(\delta)+\boldsymbol{\mathcal{A}}_{\boldsymbol{z}}-diag(\boldsymbol{s}^{*}_{t+1})diag\left(\boldsymbol{\mathcal{A}}_{\boldsymbol{jj,t+1}}f(\boldsymbol{\theta}_{\boldsymbol{t+1}})\right)$$
$$+\left(\beta^{+}G^{+}(\Delta^{+}\hat{\boldsymbol{J}}_{\boldsymbol{t}})\mathbf{1}\{\hat{w}^{*}_{b,t+1}>\hat{w}\}+\beta^{-}G^{-}(\Delta^{-}\hat{\boldsymbol{J}}_{\boldsymbol{t}})\mathbf{1}\{\hat{w}^{*}_{b,t+1}<\hat{w}\}\right)\left[\boldsymbol{\mathcal{A}}_{\boldsymbol{b,t+1}}-\boldsymbol{I}\right].$$

This is a LCP with $\zeta\equiv\hat{\boldsymbol{J}}$, $M\equiv\left(\hat{\rho}+\frac{1}{\Delta t}\right)\boldsymbol{I}-\mathbf{1}\{\hat{\boldsymbol{w}}\in\hat{\mathcal{Z}}^{h}_{t+1}\}\boldsymbol{\mathcal{A}}_{\boldsymbol{J,t+1}}$, $q\equiv-\mathbf{1}\{\hat{\boldsymbol{w}}\in\hat{\mathcal{Z}}^{h}_{t+1}\}\left(1-e^{\hat{\boldsymbol{w}}}+\frac{\hat{\boldsymbol{J}}_{\boldsymbol{t+1}}}{\Delta t}\right)$.

3.2 Update value when unemployed:

$$\hat{w}^{*}_{u,t+1}=\arg\max_{\hat{w}_u}\theta_{t+1}(\hat{w}_u)^{1-\alpha}\hat{H}_{t+1}(\hat{w}_u)$$

3.3 Solve for $\hat{U}_t$ from:

$$\hat{\rho}\hat{U}_t=\tilde{B}+\theta_{t+1}(\hat{w}^{*}_{u,t+1})^{1-\alpha}\hat{H}_{t+1}(\hat{w}^{*}_{u,t+1})+\frac{\hat{U}_{t+1}-\hat{U}_t}{\Delta t}$$

3.4 Iterate steps 3.1 to 3.4 until convergence.

4. Go back to step 3 to solve the values for period $t-1$.

A.3.1 Solving the Kolmogorov Forward Equation during the Transition

Problem: Find the secuence $g^h(\hat{w}, t)$ and $g^u(\hat{w}, t)$ such that:

$$
\begin{aligned}
\frac{\partial g^h(\hat{w},t)}{\partial t} &= -(-\gamma)\frac{\partial g^h(\hat{w},t)}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 g^h(\hat{w},t)}{\partial \hat{w}^2} - \Big(\delta + s_t(\hat{w}) f(\theta_t(\hat{w}^*_{jj,t}(\hat{w}))) + \tilde{\beta}_t(\hat{w})\Big)\, g^h(\hat{w},t) \\
&+ \int \Big[1(\hat{w}^*_{jj,t}(x) = \hat{w}) s_t(x) f(\theta_t(\hat{w})) + 1(\hat{w}^*_{b,t}(x) = \hat{w})\tilde{\beta}_t(x)\Big]\, g^h(x,t)\, \mathrm{d}x \quad \forall \hat{w} \in (\hat{w}^-_t, \hat{w}^+_t)\backslash\{\hat{w}^*_{u,t}\}, \\
\frac{\partial g^u(\hat{w},t)}{\partial t} &= -(-\gamma)\frac{\partial g^u(\hat{w},t)}{\partial \hat{w}} + \frac{\sigma^2}{2}\frac{\partial^2 g^u(\hat{w},t)}{\partial \hat{w}^2} - f(\theta_t(\hat{w}^*_{u,t}))g^u(\hat{w}) \quad \forall \hat{w} \in \mathbb{R}\backslash\{\hat{w}^*_{u,t}\}.
\end{aligned}
$$

and

$$
\begin{aligned}
g^h(\hat{w}^-_t, t) &= g^h(\hat{w}^+_t, t) = 0, \\
\lim_{\hat{w}\to-\infty} g^u(\hat{w}, t) &= \lim_{\hat{w}\to\infty} g^u(\hat{w}, t) = 0, \\
1 &= \int_{-\infty}^{\infty} g^u(\hat{w}, t)\, \mathrm{d}\hat{w} + \int_{\hat{w}^-}^{\hat{w}^+} g^h(\hat{w}, t)\, \mathrm{d}\hat{w}, \\
\frac{\partial \mathcal{E}_t}{\partial t} &= f(\theta_t(\hat{w}^*_{u,t}))(1-\mathcal{E}_t) - \delta\mathcal{E}_t - \frac{\sigma^2}{2}\left[\lim_{\hat{w}\downarrow\hat{w}^-_t}\frac{\partial g^h(\hat{w},t)}{\partial \hat{w}} - \lim_{\hat{w}\uparrow\hat{w}^+_t}\frac{\partial g^h(\hat{w},t)}{\partial \hat{w}}\right],
\end{aligned}
$$

where $\tilde{\beta}(x) := \Big(\beta^+ G^+(\Delta^+ \hat{\boldsymbol{J}}_t)\mathbf{1}\{\hat{w}^*_{b,t} > x\} + \beta^- G^-(\Delta^- \hat{\boldsymbol{J}}_t)\mathbf{1}\{\hat{w}^*_{b,t} < x\}\Big)$. To solve this, we can discretize each equation using the implicit method. Let

$$
\begin{aligned}
\boldsymbol{\mathcal{A}_t} \equiv & -diag(\delta) + \boldsymbol{\mathcal{A}_{z,t}} + diag(\boldsymbol{s}^*_t)\left[\boldsymbol{\mathcal{A}_{jj,t}} diag(f(\boldsymbol{\theta_t})) - diag\left(\boldsymbol{\mathcal{A}_{jj,t}} f(\boldsymbol{\theta_t})\right)\right] \\
& + \Big(\beta^+ G^+(\Delta^+ \hat{\boldsymbol{J}}_t)\mathbf{1}\{\hat{w}^*_{b,t} > \hat{w}\} + \beta^- G^-(\Delta^- \hat{\boldsymbol{J}}_t)\mathbf{1}\{\hat{w}^*_{b,t} < \hat{w}\}\Big)\left[\boldsymbol{\mathcal{A}_{b,t}} - \boldsymbol{I}\right],
\end{aligned}
$$

Let $\boldsymbol{I}^t_x$ be the diagonal matrix with entries equal to one if condition x is satisfied in period t. Then, the above equations can be expressed as

$$
\begin{bmatrix}
\boldsymbol{I}^t_{\hat{w}_j \in (\hat{\mathcal{W}}^{j*}_t \cap \hat{\mathcal{W}}^{h*}_t)\backslash\{\hat{w}^*_{u,t}\}}\left[\boldsymbol{I} - \Delta t \mathcal{A}^T_t\right] + \boldsymbol{I}_{\hat{w}_j \notin (\hat{\mathcal{W}}^{j*}_t \cap \hat{\mathcal{W}}^{h*}_t)\backslash\{\hat{w}^*_{u,t}\}} & \mathbf{0} \\
\mathbf{0} & \boldsymbol{I}^t_{\hat{w}_j \neq \hat{w}^*_{u,t}}\left[\boldsymbol{I} - \Delta t\left[\boldsymbol{\mathcal{A}}^T_{\boldsymbol{z,t}} - f(\theta_t(\hat{w}^*_{u,t}))\boldsymbol{I}\right]\right] \\
[\Delta w, \ldots, \Delta w] & [\Delta w, \ldots, \Delta w] \\
\Big((1+\Delta t\delta)\Delta_w[1,\ldots,1] + \frac{\Delta t\sigma^2}{2\Delta_w}\left[[0,\ldots,0,-1\{\hat{w}_j = \hat{w}^-_t\},1,0,\ldots,0] - [0,\ldots,0,-1,1\{\hat{w}_j = \hat{w}^+_t\},0,\ldots,0]\right]\Big) & -\Delta t f(\theta_t(\hat{w}^*_{u,t}))\Delta_w[1,\ldots,1]
\end{bmatrix}
\begin{bmatrix}
\boldsymbol{g^h_{t+1}} \\
\boldsymbol{g^u_{t+1}}
\end{bmatrix} =
$$

where $\tilde{g}_t \equiv [\boldsymbol{I}^t_{\hat{w}_j \in (\hat{\mathcal{W}}^{j*}_t \cap \hat{\mathcal{W}}^{h*}_t)\backslash\{\hat{w}^*_{u,t}\}}\boldsymbol{g^h_t} + \boldsymbol{I}_{\hat{w}_j \notin (\hat{\mathcal{W}}^{j*}_t \cap \hat{\mathcal{W}}^{h*}_t)\backslash\{\hat{w}^*_{u,t}\}}\mathbf{0};\ \boldsymbol{I}^t_{\hat{w}_j \neq \hat{w}^*_{u,t}}\boldsymbol{g^u_t};\ 1;\ \Delta_w[1,\ldots,1]\boldsymbol{g^h_t}]$. Note that the boundary conditions for $\boldsymbol{g^u}$ are not included in the above equations, because they are replaced by the reflecting barrier in $\boldsymbol{\mathcal{A}_z}$.